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Efficient computation of dispersion curves for multilayered waveguides and half-spaces

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Highlights

- Exponentially convergent semi-discretization is proposed for multilayered waveguides.
- Midpoint integration with linear finite elements is key to exponential convergence.
- Finite layers are discretized using a mesh stretched into complex plane.
- Half-space is modeled using an optimally graded mesh.
- Implementation requires minimal modification to existing FE-based codes.

Abstract

Motivated by the need to compute dispersion curves for layered media in the contexts of geophysical inversion and nondestructive testing, a novel discretization approach, termed complex-length finite element method (CFEM), is developed and shown to be more efficient than the existing finite element approaches. The new approach is exponentially convergent based on two key features: unconventional stretching of the mesh into complex space and midpoint integration for evaluating the contribution matrices. For modeling the layered half-spaces of infinite depth, we couple CFEM with the method of perfectly matched discrete layers (PMDL) to minimize the errors due to mesh truncation. A number of numerical examples are used to investigate the efficiency of the proposed methods. It is shown that the suggested combination of CFEM and PMDL drastically reduces the number of elements, while requiring minor modifications to the existing finite element codes. It is concluded that the methods' exponential convergence and sparse computation associated with linear finite elements, result in significant reduction in the overall computational cost.

Keywords: Guided waves; Surface waves; Layered media; Dispersion curve; Thin layer method; Perfectly matched layers

1. Introduction

Propagation of guided waves in stratified media can be exploited for obtaining the structure information in a wide range of applications. One major group of waveguides includes layered half-spaces where seismic surface waves

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including Rayleigh and Love waves can be generated and propagated near the surface. Rayleigh waves can be easily excited and recorded along a free surface since they carry the near-surface wavefield energy [1] with large amplitudes and thus high signal to noise ratio [2]. Being sensitive to the surface topography, Rayleigh waves are widely employed in the areas of near surface geophysics [3–6], pavement structures characterization [7,8] as well as geotechnical site characterization [9]. Love waves can also be exploited in near-surface inversion due to their simpler dispersion curves and higher signal-to-noise ratio and less dependency on initial models [10–12]. Furthermore the joint analysis of Rayleigh and Love waves through defining bi-objective problems can improve the predictions and reduce the uncertainties [13–15]. Lamb waves form another significant category of guided waves which exist in structures such as plates and beams, and are utilized for nondestructive evaluations such as identification of cracks in beams and slabs [16–19], damage detection in composite laminates [20] and thickness prediction of oil and gas pipelines [21].

The aforementioned inverse problems are often solved through an optimization procedure that involves multiple forward solutions, specifically the computation of dispersion curves for multilayered waveguides or half-spaces. Existing methods to calculate the dispersion curves for layered media consist of the transfer (propagator) matrix method, the stiffness matrix method and the thin layer method. Transfer matrix method was proposed by Thomson and Haskell [22,23]. This method involves a challenging root finding procedure for the highly nonlinear dispersion relation (the roots are in general complex-valued in the presence of material or radiation damping). There are also some issues associated with high frequencies and various improvements have been made so far [24–29], but many inconveniences still remain, which are mentioned in [30]. Stiffness matrix method is also based on closed form trigonometric expressions with some advantages over the transfer matrix method as discussed in [31].

Thin layer method (TLM) [32,33] is based on the finite element discretization in the transverse (heterogeneous) direction while using analytical solutions in the homogeneous directions. This method approximates the transcendental functions in the stiffness matrix with algebraic functions. However in order to achieve the required accuracy, sufficient number of elements per wave length should be adopted, leading to increase in computational cost. While the TLM was originally proposed using linear elements, an example of its high-order variant can be found in [34].

This paper presents a fast and accurate forward solution procedure that can be used for both local and global inversion algorithms. Our method is similar to TLM in that it uses finite element semi-discretization, but with two important differences. Firstly, the bounded layers are discretized with the help of recently proposed Complex-length Finite Element Method (CFEM), which has exponential convergence and requires much fewer finite elements [35]. Secondly, the half-space is also modeled with an efficient discretization technique based on Perfectly Matched Discrete Layers (PMDL), which is proven to be an effective method for modeling unbounded domains (see e.g. [36]). We show that the combination of CFEM and PMDL results in orders of magnitude reduction in the cost of computing the dispersion curves.

The rest of the paper is organized as follows. Section 2 contains a summary of the proposed algorithms. In Section 3, we discuss the model problem and governing equations for multilayered waveguides, followed by the description of standard finite-element based technique to compute the dispersion curves. Sections 4 and 5 formulate the CFEM and PMDL methods respectively. In Section 6, various numerical examples are presented to illustrate the efficiency of the proposed techniques.

2. Overview of the proposed algorithms

Consider the multilayered waveguide and half-space shown in Fig. 1(a) and (b) respectively; the objective is either the solution at layer interfaces, or the wave dispersion relations in the horizontal direction. To this end, we propose an unconventional discretization in the vertical direction, which includes two main parts: (a) discretization of finite layers using the Complex-length Finite Element Method (CFEM), and (b) discretization of the half-space using Perfectly Matched Discrete Layers (PMDL). In what follows, we briefly summarize the two approaches and elaborate and justify them in the rest of the paper.

The basic idea of CFEM is to discretize each (homogeneous) layer with regular piecewise linear finite element mesh, but with two important modifications: (a) midpoint integration is used to compute the element contribution matrices, and (b) the element depths are chosen to be specially computed complex (conjugate) values in Table 1, resulting in unconventional stretching of the finite element mesh into the complex plane. These two key modifications of the mesh result in exponential convergence of the solution with respect to number of finite elements. The implication