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Yueqiang Shang, Jin Qin

PII: S0045-7825(15)00366-7
DOI: http://dx.doi.org/10.1016/j.cma.2015.11.013
Reference: CMA 10757

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date: 10 September 2015
Revised date: 3 November 2015
Accepted date: 10 November 2015

A finite element variational multiscale method based on two-grid discretization for the steady incompressible Navier-Stokes equations ☆

Yueqiang Shang[a,*], Jin Qin[b]

[a]School of Mathematics and Statistics, Southwest University, Chongqing 400715, P. R. China
[b]School of Mathematics and Computer Science, Zunyi Normal college, Zunyi 563002, P. R. China

Abstract

By combining the best algorithmic features of two-grid discretization method and a recent variational multiscale method, a two-level finite element variational multiscale method based on two local Gauss integrations for convection dominated incompressible Navier-Stokes equations is proposed and analyzed. In this method, a fully nonlinear Navier-Stokes problem is first solved on a coarse grid, and then a linear problem is solved on a fine grid to correct the coarse grid solution, where the numerical forms of the Navier-Stokes equations both on coarse and fine grids are stabilized by a stabilization term defined by the difference between a consistent and an under-integrated matrix of the velocity gradient. Error bounds of the approximate solution are analyzed. Algorithmic parameter scalings of the method are derived. Numerical tests are also given to verify the theoretical predictions and demonstrate the efficiency and promise of the method.

Keywords: Navier-Stokes equations, finite element, variational multiscale method, two-grid method

1. Introduction

For an incompressible Newtonian viscous fluid, the governing equations of its flow are the Navier-Stokes equations. It is generally accepted that incompressible fluid flows, even for high Reynolds numbers, are faithfully modeled by the Navier-Stokes system. Therefore, a computational fluid dynamics model is in general based on the solution of the Navier-Stokes equations and its discretization scheme such as finite element methods, finite volume methods and finite difference methods. As a result, many attentions were put on the development of efficient numerical methods for the Navier-Stokes equations in the last decades; see, for example, the monographs of Temam [1], Girault and Raviart [2, 3], and Elman, Silvester and Wathen[4], among others.

Among the successful discretization methods, two-level or multilevel finite element methods get popular since the pioneered work of Xu [5, 6]. For examples, Layton et al. [7, 8, 9, 10, 11], Girault and Lions [12], He et al. [13, 14, 15, 16], Liu and Hou [17, 18] have studied two-level or multilevel methods for the stationary Navier-Stokes

☆This work was supported by the Natural Science Foundation of China (No. 11361016), the Project Sponsored by the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry, Fundamental Research Funds for the Central Universities (No. XDJK2014C160, SWU113095), and the Science and Technology Foundation of Guizhou Province, China (No. [2013]2212).

*Corresponding author. Tel. +86-23-68252350. E-mail: yqshang@swu.edu.cn