Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 300 (2016) 461–489
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Construction and application of an ellipsoidal convex model using a semi-definite programming formulation from measured data

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Received 20 April 2015; received in revised form 23 November 2015; accepted 24 November 2015
Available online 30 November 2015

Abstract

As a set theory-based convex model, the ellipsoidal model provides an attractive framework for treating uncertain-but-bounded variations in the structural reliability analysis and design optimization. However, improper modeling of the uncertainties may give rise to misleading non-probabilistic reliability analysis, thus result in either unsafe or over-conservative designs. This paper presents a systematic study on the mathematical formulation for constructing the minimum-volume ellipsoidal convex model using a given set of sample data, and shows its application in existing methods of non-probabilistic reliability analysis and design optimization of structures with bounded uncertainties. In this method, the uncertain parameters are first divided into groups according to their sources. For each individual group of uncertainties, the minimum-volume ellipsoid problem is reformulated into a semi-definite programming (SDP) problem and thus can be efficiently solved to its global optimum. Further, a linear transformation based on the eigenvalue analysis is employed to map the ellipsoidal model into a standard uncertainty space. This uncertainty modeling technique enables a compact and differentiable bound description of the parameter variations. Moreover, it has another useful property, the affine invariance, which is shown to be necessary for meaningful definition of a non-probabilistic reliability index. The effectiveness and efficiency of the present techniques for convex model construction and the corresponding reliability analysis are demonstrated with numerical examples of structural topology optimization problems with bounded variations arising from different sources.
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Keywords: Bounded uncertainty; Non-probabilistic model; Convex model; SDP; Structural reliability; Reliability-based topology optimization

1. Introduction

Inherent uncertainty may greatly degrade structural performance. Along with the trend to push engineering structural designs towards their limits of mechanical performance, there has been an ever increasing demand for reliable uncertainty analysis of structures to ensure their functionality and safety during service life [1].

The mathematical models currently available for various uncertainties in the structural analysis and design can be broadly classified into probabilistic and non-probabilistic models [2]. As the most mathematically mature uncertainty

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http://dx.doi.org/10.1016/j.cma.2015.11.025
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model, the former describes the uncertain parameters with random fields or discrete random variables that have certain statistical distribution characteristics, which are usually extracted from measured data or assumed on the basis of engineering experiences. The probabilistic model has been widely used in many real engineering applications for structural reliability-based design optimization (RBDO) [3,4]. However, as shown by Ben-Haim [5], unjustified assumptions on the probabilistic distribution may lead to large errors in the structural reliability prediction. This means that the traditional probabilistic approaches might be questionable to deal with those problems involving incomplete information or inherently non-probabilistic uncertainties. As supplements to traditional probability models, non-probabilistic models (including the convex model, interval model and fuzzy sets) have drawn considerable attention both in academic research and engineering applications [2,6]. The convex models describing variation bounds of the input parameters are particularly useful when no exact probability distribution information but only bounds of input parameters are available, or when these parameters are in nature bounded ones (e.g. manufacturing errors under tolerance band control) [7]. Recently, some variants of traditional convex models have also been introduced, including the parallelepiped model [8], the super-ellipsoidal model [9] and the convex model process [10]. Comparative studies have been carried out on constructing different convex models with available experiment data [11,12]. Among these models, the ellipsoidal convex model provides a smooth and differentiable bound description of uncertainties and thus greatly facilitates the extreme analysis and design optimization with gradient-based mathematical programming algorithms.

Though structural analysis and design optimization incorporating interval or ellipsoidal convex models of bounded uncertainties have been extensively studied [13–21], how to construct ellipsoidal convex models in an elegant and efficient way has been surprisingly rarely addressed and still remains a major challenge greatly hindering practical applications. In nearly all the studies on non-probabilistic reliability analysis [22,23] and design optimization [17,19], the used ellipsoidal models are presumed. Ben-Tal and Nemirovski [24] constructed the ellipsoid model enclosing multiple load cases under a strong assumption that the ellipsoid center is located at the origin of the uncertainty space. Up to now, two numerical methods have been developed to construct the ellipsoidal models in a general setting. Zhu et al. [25] presented an approach (referred to as the rotation matrix method hereafter) to seek the minimum-volume ellipsoid with the same direction and center as the coordinate bounding box of all experimental samples in each rotated coordinate system. All possible directions of the ellipsoid must be exhaustively searched in this method, which means that the computation time increases exponentially with the dimension and this procedure may become extremely expensive as the number of uncertain variables grows. Also, the center of the ellipsoid are restricted to that of the coordinate bounding box. Therefore, the authors pointed out that the search for minimum-ellipsoid "cannot be executed this way" even when only 11 parameters were considered since the computation would cost "3.1×10^6 years" [26]. More recently, Jiang et al. [27] proposed a method based on correlation analysis, with which one can construct multidimensional ellipsoidal convex models using the covariance information. However, as shown in their study, this method cannot ensure that all the sample points are enclosed in the constructed ellipsoids (see Fig. 8 in [27]). Both methods appear to be effective for some academic problems, but their numerical performance in high-dimensional uncertainty space is not guaranteed. Therefore, it is still highly desired to develop a more general framework with mathematical rigor and high efficiency, for constructing the ellipsoidal models from a given set of measured data. An accurate ellipsoid modeling method for bounded uncertainties can directly enhance the confidence level of the non-probabilistic reliability analysis and optimization. Moreover, some fundamental issues related to the convex set modeling, e.g. the affine invariance property and the objectivity, have yet to be exposed.

Towards this end, this paper aims to develop a systematic and efficient mathematical formulation for constructing compact ellipsoidal models using measured data of a structural system, as well as numerical treatment for non-probabilistic reliability analysis and design optimization. We consider an equivalent semi-definite programming (SDP) formulation for determining the minimum-volume ellipsoids enclosing all the data points in the uncertainty space. Such a problem can be readily solved to its global optimum using a standard SDP optimizer. The transformation of dimensionless uncertainty space and the mathematical definition of structural reliability index based on multi-ellipsoid convex set are also discussed.

The rest of the paper is organized as follows. In Section 2, we introduce the basic concept of ellipsoidal convex model and an SDP formulation for constructing the minimum-volume ellipsoid model of multi-source bounded uncertainties using available sample data. Then the affine invariance property of such a model is discussed. Section 3 is devoted to mathematical statements of a non-probabilistic reliability index and the corresponding reliability-based topology optimization problems based on the multi-ellipsoid convex model. Theoretical results on some important

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