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An inverse model and mathematical solution for inferring viscoelastic properties and dynamic deformations of heterogeneous structures

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ABSTRACT

We proposed an adjoint-based inverse method and mathematical solutions using Lagrange multiplier theorem for inferring dynamic viscoelastic properties of heterogeneous structures to improve computation reliability and efficiency. Existing methods such as the one using finite-difference approximated gradient may not be efficient and accurate enough for considering complex situations of the coupled effects of dynamic loading, material deformation memory, and structural heterogeneity. The proposed method derives both gradient and Hessian mathematically to satisfy the first-order necessary and second-order sufficient optimal conditions, which has great advantages compared to other approaches. We also proposed robust numerical algorithms for accurate and fast computations, including a regularization to control reasonable parameter ranges, a Newton's method with Hessian function to determine search direction for fast convenience, and a modified Armijo rule to find a stable step length. We developed a Galerkin time-domain finite-element method for numerical solutions of resulting partial differential equations, and validated the model for a layered structure. Results indicate that the proposed method has greatly improved computation accuracy and speed as compared to existing approaches. It may be applied to a broad range of heterogeneous materials and structures at different length and time scales.

Key words: Viscoelastic material, optimization, finite elements, dynamics, structures

Nomenclature

$\boldsymbol{u}$: displacement	$\mathbf{E}$: Young's modulus
u: displacement discretized at nodes	$\boldsymbol{R}$: relaxation modulus matrix
$\hat{\boldsymbol{u}}$: variation of $\boldsymbol{u}$	$\mathcal{L}$: lagrangian
$\tilde{\boldsymbol{u}}$: incremental variation of $\boldsymbol{u}$	g: gradient, $\mathrm{g} = \partial\mathcal{L}/\partial\mathbf{m}$
$\boldsymbol{p}$: test function or Lagrangian multiplier	$\mathbf{H}$: hessian, $\mathbf{H} = \partial^2\mathcal{L}/\partial\mathbf{m}^2$
p: joint discretized at nodes	H: hessian discretized at nodes
$\hat{\boldsymbol{p}}$: differential of $\boldsymbol{p}$	f: loading
$\tilde{\boldsymbol{p}}$: Incremental variation of $\boldsymbol{p}$	b: body force
$\mathbf{m}$: material model parameter	$\boldsymbol{\sigma}$: stress tensor
m: material parameter vector discretized at nodes	$\mathbf{M}$: mass matrix;
$\mathbf{m}^*$: pre-defined material model parameter	$\mathbf{C}$: damping matrix
$\hat{\mathbf{m}}$: variation of $\mathbf{m}$	$\boldsymbol{\Phi}$: shape function matrix
$\tilde{\mathbf{m}}$: incremental variation of $\mathbf{m}$	$\mathbf{B}$: strain-displacement matrix, $\mathbf{B} = \nabla\boldsymbol{\Phi}$
$E(t)$: relaxation modulus	$\boldsymbol{\nabla}$: gradient, $\boldsymbol{\nabla} = \frac{\partial}{\partial x_1} i + \frac{\partial}{\partial x_2} j + \frac{\partial}{\partial x_3} k$
$E^*(f)$: dynamic modulus	$\boldsymbol{\nabla}\cdot$: divergence, $\nabla\cdot \mathrm{f} = \frac{\partial \mathrm{f}_1}{\partial \mathrm{x}_1} + \frac{\partial \mathrm{f}_2}{\partial \mathrm{x}_2} + \frac{\partial \mathrm{f}_3}{\partial \mathrm{x}_3}$
E_i: elastic modulus of springs of Maxwell model	$\forall$: for all
η_i: viscosity of dashpots of Maxwell model	