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A spectral element formulation of the immersed boundary method for Newtonian fluids ☆

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Abstract

A spectral element formulation of the immersed boundary method (IBM) is presented. The spectral element formulation (SE-IBM) is a generalisation of the finite element immersed boundary method (FE-IBM) based on high-order approximations of the fluid variables. Several schemes for tracking the movement of the immersed boundary are considered and a semi-implicit Euler scheme is shown to offer advantages in terms of accuracy and efficiency. High-order spectral element approximations provide improved area conservation properties of the IBM due to the incompressibility constraint being more accurately satisfied. Superior orders of convergence are obtained for SE-IBM compared with FE-IBM in both L^2 and H^1 norms. The area conservation and convergence properties of the scheme are demonstrated on a series of benchmark problems.

MSC: 65M70; 74F10

Keywords: Immersed boundary method; Spectral element; Newtonian; Fluid–structure interaction

1. Introduction

In a classical formulation of a fluid–structure interaction problem, the fluid and structure are treated separately where the fluid is solved on a time-dependent domain and coupled to the structure equations using appropriately chosen interface conditions. The fluid–structure system of equations is then solved computationally using either a partitioned approach or a monolithic approach. A monolithic approach involves solving a single non-linear system of equations for both the fluid and the structure. A partitioned approach involves two systems of equations which are solved separately and then coupled together by interface conditions. A common approach in the literature is to formulate the fluid equations using the Arbitrary–Lagrangian–Eulerian (ALE) technique (see e.g. [1,2]). One of the major drawbacks of the classical approach is the computational time required — remeshing is often needed as the computational domain for the fluid equations is time-dependent. ALE was introduced to overcome the difficulties

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caused by the reconstruction of the mesh in time. In a fluid–structure interaction problem, the fluid is considered in an ALE formulation and the structure in a Lagrangian formulation [2]. The ALE formulation introduces an additional frame of reference, called the referential frame or configuration, which tracks the motion of the mesh. The classical approach to fluid–structure interaction problems using an ALE formulation is quite complex, particularly the interface conditions which have to be formulated using the so-called ALE map. Additionally, an ALE formulation can be computationally expensive when large deformations are considered. An alternative approach was introduced by Peskin and is called the immersed boundary method (IBM).

The immersed boundary method (IBM), proposed by Peskin for studying flow patterns around heart valves [3], has been applied to a wide range of problems including arterial blood flow [4], modelling of the cochlea [5], modelling of red blood cells in Poiseuille flow [6] and flows involving suspended particles [7]. A comprehensive list of applications can be found in [8]. The IBM is both a mathematical formulation and a numerical scheme for fluid–structure interaction problems. As mentioned above, in a classical fluid–structure interaction problem, the fluid and the structure are considered separately and then coupled together via some suitable jump conditions. In the IBM however, the structure – which is usually immersed in a Newtonian fluid – is viewed as being part of the surrounding fluid. This means that only a single equation of motion needs to be solved (i.e. a one-phase formulation). Additionally, the IBM allows the immersed structure to move freely over the underlying fluid mesh, alleviating the need for the remeshing required in a classical formulation.

The IBM replaces the immersed structure with an Eulerian force distribution. This Eulerian force distribution is calculated by *spreading* a Lagrangian force density to the underlying fluid using the Dirac delta distribution. The position of the immersed structure is then automatically tracked in an *interpolation* phase, where the local fluid velocity is interpolated onto the immersed structure using the Dirac delta distribution. For numerical computations, a smoothed approximation of the delta distribution is required and the same approximation must be used for both the spreading and the interpolation phases.

The original IBM proposed by Peskin [3] is based on a couple of assumptions: the immersed structure is fibrous and the viscosity is constant throughout the computational domain. While the first assumption may be physically realistic in certain cases, the second assumption is in general not desirable. The immersed finite element method (IFEM) proposed by Zhang et al. [9] used finite elements for both the fluid and the immersed structure. Using finite elements for the structure alleviates the first assumption in the original IBM and allows for a more physically realistic representation of a thick immersed structure. Additionally, IFEM used the Reproducing Kernel Particle Method (RKPM) to construct an approximation to the Dirac delta distribution. The approximation used in the original IBM is C^1 continuous. However, the approximation constructed from RKPM is C^N continuous as RKPM allows the exact reconstruction of polynomials of degree N. The finite element immersed boundary method (FE-IBM), proposed by Boffi and Gastaldi [10], also overcomes the first assumption of the original IBM and this is the method adopted in this paper. Like IFEM, the FE-IBM uses finite elements for both the fluid and the immersed structure. However the key difference between the two methods is that FE-IBM does not numerically approximate the Dirac delta distribution. Instead, the interaction is governed within a weak formulation using the action of the delta distribution on a test function and using the sifting property of the delta function. Both the IFEM and FE-IBM, suffer from the limitation of constant viscosity throughout the computational domain.

In this paper, we apply a high-order method to the FE-IBM, which we call the spectral element immersed boundary method (SE-IBM). The aim of using a high-order method is to improve the accuracy of the spreading and interpolation phases and thus improve the order of convergence of the velocity and pressure variables.

This paper is constructed as follows: Sections 2 and 3 are concerned with the fluid equations and the derivation of the FE-IBM. Section 4 discusses the spatial discretisation and Section 5 summarises the temporal stability properties of the SE-IBM. Section 6 illustrates the area conservation of the SE-IBM and finally Section 7 illustrates the application of the SE-IBM to some well known benchmark problems. Section 8 presents our conclusions and discusses avenues for future work.

2. Newtonian fluid

Let Ω_t^f, $t \in (t_0, T]$ (where t_0 and T are the initial and final times respectively), be the time-dependent fluid domain. The equations governing the motion of an incompressible fluid flow can be characterised by the incompressible