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MIXED FINITE ELEMENT-BASED FULLY CONSERVATIVE METHODS FOR SIMULATING WORMHOLE PROPAGATION*

JISHENG KOU†, SHUYU SUN‡, AND YUANQING WU§

Abstract. Wormhole propagation during reactive dissolution of carbonates plays a very important role in the product enhancement of oil and gas reservoir. Because of high velocity and nonuniform porosity, the Darcy–Forchheimer model is applicable for this problem instead of conventional Darcy framework. We develop a mixed finite element scheme for numerical simulation of this problem, in which mixed finite element methods are used not only for the Darcy-Forchheimer flow equations but also for the solute transport equation by introducing an auxiliary flux variable to guarantee full mass conservation. In theoretical analysis aspects, based on the cut-off operator of solute concentration, we construct an analytical function to control and handle the change of porosity with time; we treat the auxiliary flux variable as a function of velocity and establish its properties; we employ the coupled analysis approach to deal with the fully coupling relation of multivariables. From this, the stability analysis and *a priori* error estimates for velocity, pressure, concentration and porosity are established in different norms. Numerical results are also given to verify theoretical analysis and effectiveness of the proposed scheme.

Key words. Mixed finite element methods; Wormhole; Error estimate; Darcy-Forchheimer model.

AMS subject classifications. 65M60; 65M12; 65M15

1. Introduction. Matrix acidization technique plays an important role in enhancing oil production rate when muds and fines deposit at the perforated well pore pipe [4, 5, 7, 9, 15, 20, 26, 29, 30]. In the technique, acid is injected into matrix to dissolve the rocks and deposits around the well bore and a channel with high porosities is formed. Such channel is called wormhole. Oil and gas components in the reservoir can be pushed to the surface easily through the channel. In the procedure, the solute transport equation can be expressed as below [20]

$$\frac{\partial(\phi c_f)}{\partial t} + \nabla\cdot(\mathbf{u}c_f) - \nabla\cdot(\phi\mathbf{D}_e\nabla c_f) = k_c a_v(c_s - c_f) + f_P c_f + f_I c_I, \qquad (1.1)$$

where c_f is the cup-mixing concentration of the acid in the fluid phase, ϕ is the porosity, t is the time, $\mathbf{u}$ is the velocity, $\mathbf{D}_e$ is the effective dispersion tensor, k_c is the local mass-transfer coefficient, a_v is the interfacial area available for reaction per unit volume of the medium. The functions f_P and f_I are production and injection rates respectively, and c_I is the injected concentration. The variable c_s is the concentration of the acid at the fluid-solid interface, and the relationship between c_f and c_s is shown as

$$c_s = \frac{c_f}{1 + k_s/k_c}, \qquad (1.2)$$

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