## **Accepted Manuscript**

Mixed finite element-based fully conservative methods for simulating wormhole propagation

Jisheng Kou, Shuyu Sun, Yuanqing Wu

 PII:
 S0045-7825(15)00309-6

 DOI:
 http://dx.doi.org/10.1016/j.cma.2015.09.015

 Reference:
 CMA 10710

To appear in: Comput. Methods Appl. Mech. Engrg.

Received date:4 May 2015Revised date:12 September 2015Accepted date:19 September 2015

| Volume 273, Published 1 May 2014                                     | 150×1045.7   |
|--|--|
| Computer<br>methods in<br>applied<br>mechanics<br>and<br>engineering | Editors<br>J. Alan, Hayan<br>A. Man, H., Kaba,<br>M. M. Kananan<br>Mana, Sananan<br>Mana, Sanan<br>Mana, Sanan, Sana |
| Autority article art was subscription to an<br>ScienceDirect         | htg:/www.abaniar.com/tocata/ona  |

Please cite this article as: J. Kou, S. Sun, Y. Wu, Mixed finite element-based fully conservative methods for simulating wormhole propagation, *Comput. Methods Appl. Mech. Engrg.* (2015), http://dx.doi.org/10.1016/j.cma.2015.09.015

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

## MIXED FINITE ELEMENT-BASED FULLY CONSERVATIVE METHODS FOR SIMULATING WORMHOLE PROPAGATION\*

JISHENG KOU<sup>†</sup>, SHUYU SUN<sup>‡</sup>, AND YUANQING WU<sup>§</sup>

Abstract. Wormhole propagation during reactive dissolution of carbonates plays a very important role in the product enhancement of oil and gas reservoir. Because of high velocity and nonuniform porosity, the Darcy–Forchheimer model is applicable for this problem instead of conventional Darcy framework. We develop a mixed finite element scheme for numerical simulation of this problem, in which mixed finite element methods are used not only for the Darcy-Forchheimer flow equations but also for the solute transport equation by introducing an auxiliary flux variable to guarantee full mass conservation. In theoretical analysis aspects, based on the cut-off operator of solute concentration, we construct an analytical function to control and handle the change of porosity with time; we treat the auxiliary flux variable as a function of velocity and establish its properties; we employ the coupled analysis approach to deal with the fully coupling relation of multivariables. From this, the stability analysis and *a priori* error estimates for velocity, pressure, concentration and porosity are established in different norms. Numerical results are also given to verify theoretical analysis and effectiveness of the proposed scheme.

Key words. Mixed finite element methods; Wormhole; Error estimate; Darcy-Forchheimer model.

AMS subject classifications. 65M60; 65M12; 65M15

1. Introduction. Matrix acidization technique plays an important role in enhancing oil production rate when muds and fines deposit at the perforated well pore pipe [4,5,7,9,15,20,26,29,30]. In the technique, acid is injected into matrix to dissolve the rocks and deposits around the well bore and a channel with high porosities is formed. Such channel is called wormhole. Oil and gas components in the reservoir can be pushed to the surface easily through the channel. In the procedure, the solute transport equation can be expressed as below [20]

$$\frac{\partial(\phi c_f)}{\partial t} + \nabla \cdot (\mathbf{u}c_f) - \nabla \cdot (\phi \mathbf{D}_e \nabla c_f) = k_c a_v (c_s - c_f) + f_P c_f + f_I c_I, \quad (1.1)$$

where  $c_f$  is the cup-mixing concentration of the acid in the fluid phase,  $\phi$  is the porosity, t is the time, **u** is the velocity,  $\mathbf{D}_e$  is the effective dispersion tensor,  $k_c$  is the local mass-transfer coefficient,  $a_v$  is the interfacial area available for reaction per unit volume of the medium. The functions  $f_P$  and  $f_I$  are production and injection rates respectively, and  $c_I$  is the injected concentration. The variable  $c_s$  is the concentration of the acid at the fluid-solid interface, and the relationship between  $c_f$  and  $c_s$  is shown as

$$c_s = \frac{c_f}{1 + k_s/k_c},\tag{1.2}$$

<sup>\*</sup>This work is supported by National Natural Science Foundation of China (No.11301163), the Key Project of Chinese Ministry of Education (No.212109) and the KAUST research fund.

<sup>&</sup>lt;sup>†</sup>School of Mathematics and Statistics, Hubei Engineering University, Xiaogan 432000, Hubei, China.

<sup>&</sup>lt;sup>‡</sup>Corresponding author. Computational Transport Phenomena Laboratory, Division of Physical Science and Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, Kingdom of Saudi Arabia. Email: shuyu.sun@kaust.edu.sa.

<sup>&</sup>lt;sup>§</sup>Computational Transport Phenomena Laboratory, Division of Physical Science and Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, Kingdom of Saudi Arabia. Email: yuanqing.wu@kaust.edu.sa.

Download English Version:

https://daneshyari.com/en/article/6916598

Download Persian Version:

https://daneshyari.com/article/6916598

Daneshyari.com