Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 294 (2015) 19–55
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

High-order mortar-based element applied to nonlinear analysis of structural contact mechanics

A.P.C. Dias[a], A.L. Serpa[b], M.L. Bittencourt[a,*]

[a] Department of Integrated Systems, Faculty of Mechanical Engineering, University of Campinas, P.O. Box 6122, Zip Code 13083-860 Campinas, SP, Brazil

[b] Department of Computational Mechanics, Faculty of Mechanical Engineering, University of Campinas, P.O. Box 6122, Zip Code 13083-860 Campinas, SP, Brazil

Received 25 October 2014; received in revised form 18 May 2015; accepted 20 May 2015
Available online 5 June 2015

Abstract

In this paper we present a high-order contact element using a mortar-based domain decomposition method. The main objective is to verify the solution accuracy by increasing the element interpolation order in two-dimensional contact problems, considering large deformations and friction. We present studies of accuracy, processing time and contact stress oscillation using h- and p-refinements. The comparative results show that the high-order interpolation is a strategy with superior performance for the contact problems analysed. The results for the p-refinement improved the solution accuracy of the stresses and forces generated by contact with a lower processing time.

Keywords: High-order finite element method; Contact mechanics; High-order mortar-based contact element; Large deformations; Hyperelastic materials

1. Introduction

The boundary value problems involving nonlinear contact are of great importance in various applications in mechanical and civil engineering, as well as in environmental and biomedical applications.

This paper presents the application of the high-order *hp* finite element method (*hp*-FEM) to the problem of contact mechanics. The efficiency and advantages of the *hp*-FEM have been studied in nonlinear structural problems such as elastoplasticity [1–3], powder metallurgy [4] and contact with small deformations [5,6]. There have been significant growth areas of applicability of the *hp*-FEM. The advantages over classical methods are exponential convergence rate for smooth problems [7], flexibility in using elements with large aspect ratio, locking-free with respect to the thickness of plate and shell elements and incompressible materials [8]. However, contact mechanics has received relatively less attention in the high-order literature.

* Corresponding author.
E-mail address: mlb@fem.unicamp.br (M.L. Bittencourt).

http://dx.doi.org/10.1016/j.cma.2015.05.013

The *hp*-FEM was applied in frictionless contact problems considering small deformations in [5,9,6]. High-order NTN contact elements were used in [5] and the results showed a significant gain in the solution accuracy by increasing the polynomial order. The work [9] presents an analysis of the Hertz contact problem using Legendre polynomials for the construction of interpolation functions for NTS contact elements. The results showed that the use of *hp*-FEM in this class of problems is a promising strategy, increasing the accuracy of results for contact stresses with less degrees of freedom.

The mortar method was originally proposed for domain decomposition techniques [10] and later adapted for frictionless contact problems in [11]. The method consists of elements which are built from nodes and edges of different bodies in contact. As opposed to the usual formulation of the mortar finite element, the mortar contact finite element does not join the surfaces. These elements produce only response to compression loads in the contact region for preventing a body to penetrate the other. The formulation was extended to frictional contact problems considering large deformations and low-order interpolation in [12].

Mortar and mortar-based linear elements are commonly used for the discretization of the contact terms [13–20]. Recent papers, comparing results obtained with linear and quadratic contact elements, show a gain in the solution accuracy with the use of quadratic contact elements [21,14,22,23,13,24–26].

Comparative simulations in large deformation show better accuracy applying fewer degrees of freedom using quadratic contact element [23,14]. In [23], the discretization of frictionless contact for the Lagrange multiplier and penalty methods is made with linear and quadratic interpolations. In [13], the discretization of the friction problem considers the same strategy proposed in [23]. These papers show that high-order interpolation is an effective strategy in contact problems, but only analysis with quadratic functions were presented. The papers [5,9] applied high-order Legendre polynomials for building the NTS and STS contact elements, respectively. Even obtaining significant results, these papers address problems considering small deformations. Refs. [27–30] obtained results using isogeometric high-order interpolation with NURBS in the construction of the contact element. Those papers present results for Hertz problem and show that high-order interpolation with NURBS helped to reduce contact stress oscillation in the on/off contact interface, but the results were achieved with application of very fine meshes on the contact area.

This paper presents a fully high-order mortar-based contact element for solution of two-dimensional contact problems, considering small and large deformations and friction. The Neo-Hookean isotropic compressible hyper-elastic material model is considered for large deformations. The proposed formulation is the high-order extension of the formulation developed in [23,13]. This contact element is also named as non-mortar contact element in [28] (or Gauss-point-to-segment contact element (GPTS)). It enforces the contact constraints at each of the quadrature points associated with the contact contribution. Firstly, we tested the high-order contact element in a contact patch test well-known in the literature [31]. Then, we apply this element in small and large deformations with known solutions in the literature and propose a study of the oscillatory stress profile in the on/off contact interface region. The results indicate a relationship between the choice of penalty parameter and the percentage of the element that must remain active, in order to obtain a control of stresses profile without oscillations. The results for *h*- and *p*-refinements show that high-order interpolation can be considered a strategy with superior performance in contact problems, when the objective of the analysis is to increase the accuracy of results for stresses and forces generated by contact with a lower time/cost processing.

2. Problem description and variational formulation

In this paper we consider problems involving nonlinear effects of large deformations with two or more bodies in contact. The presented formulation is based on the Refs. [32,23]. A brief overview is presented in this section.

Consider two elastic bodies $\boldsymbol{B}^\alpha$ with $\alpha = 1, 2$, each one occupying a delimited domain $\Omega^\alpha \subset R^2$, as shown in Fig. 1. The boundary Γ^α consists of three parts: Γ_σ with prescribed edge loads, Γ_u with prescribed displacements and Γ_c called contact surface where the two bodies B^1 and B^2 get in contact. The following relationships must be satisfied: $\Gamma^\alpha = \Gamma^\alpha_\sigma \cup \Gamma^\alpha_u \cup \Gamma^\alpha_c$ and $\Gamma^\alpha_\sigma \cap \Gamma^\alpha_u \cap \Gamma^\alpha_c = \emptyset$. It is considered that two distinct points of the bodies in the initial configuration, $\mathbf{X}^1$ and $\mathbf{X}^2$, can occupy the same position in the current configuration, $\mathbf{x}^2 = \varphi(\mathbf{X}^2, t) = \mathbf{x}^1 = \varphi(\mathbf{X}^1, t)$, where contact conditions have to be formulated.

The strategy commonly used to solve contact problems is to treat it as a constrained optimization problem [33,32,23]. The objective function is the total potential energy of the bodies in contact $\Pi(\mathbf{u})$, which is the sum of