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A stochastic computational multiscale approach; Application to MEMS resonators

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Highlights

- This paper develops a stochastic multiscale model for polycrystalline materials.
- At the finest scale, the material is modeled using a random Voronoï tessellation.
- Homogenization of SVEs leads to a stochastic characterization at the mesoscale.
- A random field of the meso-scale elastic tensor is then generated.
- The meso-scale uncertainties are then propagated to the structural scale using a SFEM.

Abstract

The aim of this work is to develop a stochastic multiscale model for polycrystalline materials, which accounts for the uncertainties in the micro-structure. At the finest scale, we model the micro-structure using a random Voronoï tessellation, each grain being assigned a random orientation. Then, we apply a computational homogenization procedure on statistical volume elements to obtain a stochastic characterization of the elasticity tensor at the meso-scale. A random field of the meso-scale elasticity tensor can thus be generated based on the information obtained from the SVE simulations. Finally, using a stochastic finite element method, these meso-scale uncertainties are propagated to the coarser scale. As an illustration we study the resonance frequencies of MEMS micro-beams made of poly-silicon materials, and we show that the stochastic multiscale approach predicts results in agreement with a Monte Carlo analysis applied directly on the fine finite-element model, *i.e.* with an explicit discretization of the grains.
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Keywords: Multi-scale; Stochastic; Finite elements; Polycrystalline; Resonance frequency; MEMS

1. Introduction

Uncertainty is an inherent nature of materials, especially when they are heterogeneous. The heterogeneity of a material has a significant impact on its properties and might influence the response of structures made of that material

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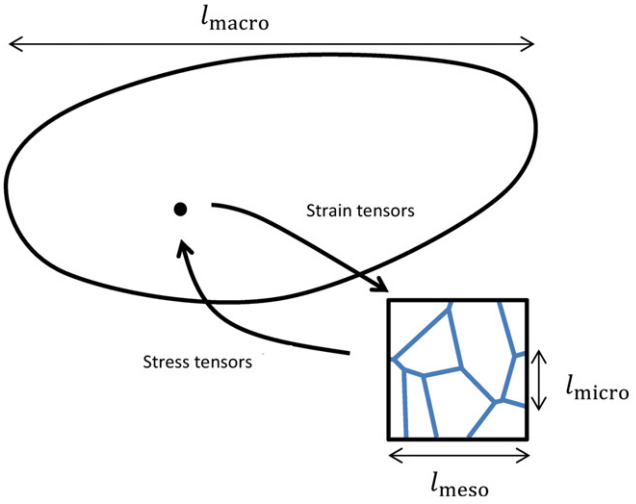

Fig. 1. The different scales involved in a multiscale analysis.

as well. The uncertainty due to the heterogeneous nature of the material can be described as the spatial variability of the material properties in the structure. When considering a finite element analysis, the structural response variability can be predicted using the direct Monte Carlo method, which can lead to an overwhelming computation cost as it involves the finite-element discretization of the heterogeneities. In order to solve the problem of structural stochasticity at a reasonable computation cost, Stochastic Finite Element (SFE) analyses were developed [1–3].

In the context of stochastic finite element analyses, a random field, which is used to describe the heterogeneity of a material, is discretized in accordance with the finite element mesh. The proper mesh size depends primarily on the standard deviation and correlation length of the random field, which constrains the variation of the random variable within each element to be small enough [4]. Therefore finite element sizes smaller than the correlation length, such as one half of the correlation length [5], are required to ensure the accuracy of the analysis results. In all generalities, the correlation length of the random material property depends on the characteristic length of the material heterogeneity. As the interest of this work focuses on polycrystalline materials, the correlation length is related to the size of the grains, meaning that each grain needs to be meshed in a conformed way. With such a method, the analysis of the structural stochasticity is very expensive in terms of computational resources. Moreover using a finite element discretization based on the explicit micro-grain structure leads to a noise field [6] instead of a smooth one [7]. Therefore the stochastic finite element method, such as the Neumann expansion [8] or perturbation approximation [9], cannot be applied. To achieve a reduction of the computational cost of a structural stochasticity analysis, we seek the recourse to multi-scale computational methods in which a smooth random field is defined at the meso-scale.

Multi-scale methods were developed with the rise of structural applications made of heterogeneous materials such as composite materials, metal alloys, and polycrystalline materials. In a multi-scale method, the macro-scale or structural-scale behavior can be related to the micro-scale properties, through a homogenization technique. The different scales involved in such an analysis, which are depicted in Fig. 1, are

- **The micro-scale**: is the characteristic size of the micro-structure, such as the size of inclusions for composite materials or the size of grains for polycrystalline materials, and is denoted by l_{micro};
- **The meso-scale**: is an intermediate scale, such as the size of the volume element over which the homogenization is performed, and is denoted by l_{meso};
- **The macro-scale**: also called structural scale, is the size of the structural problem, and is denoted by l_{macro}.

Homogenization methods relate the macroscopic strain tensor to the macroscopic stress tensor through the resolution of a meso-scale boundary value problem (BVP). In this framework, the structural-scale BVP is seen as a continuum homogeneous medium and the meso-scale BVP contains the different sources of heterogeneities. To this end, the meso-scale BVP is defined on a Representative Volume Element (RVE), which represents the micro-structure and the micro-structural behavior in a statistically representative way.

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