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Comput. Methods Appl. Mech. Engrg. 293 (2015) 38–70
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Divergence-conforming discretization for Stokes problem on locally refined meshes using LR B-splines

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Received 4 July 2014; received in revised form 27 March 2015; accepted 30 March 2015
Available online 23 April 2015

Abstract

To solve the incompressible flow problems using isogeometric analysis, the div-compatible spline spaces were originally introduced by Buffa et al. (2011), and later developed by Evans (2011). In this paper, we extend the div-compatible spline spaces with local refinement capability using Locally Refined (LR) B-splines over rectangular domains. We argue that the spline spaces generated on locally refined meshes will satisfy compatibility provided they span the entire function spaces as governed by Mourrain (2014) dimension formula. We will in this work use the *structured* refined LR B-splines as introduced by Johannessen et al. (2014). Further, we consider these div-compatible LR B-spline spaces to approximate the velocity and pressure fields in mixed discretization for Stokes problem and a set of standard benchmark tests are performed to show the stability, efficiency and the conservation properties of the discrete velocity fields in adaptive isogeometric analysis.
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Keywords: Isogeometric analysis; LR B-splines; Local refinement; Compatible spaces; De Rham complex; Stokes

1. Introduction

Isogeometric analysis (IGA) was introduced in [1] as an innovative numerical methodology for the discretization of Partial Differential Equations (PDEs). The main idea was to improve the interoperability between Computer Aided Design (CAD) and PDE solvers, and to achieve this, the authors in [1] proposed to use CAD mathematical primitives, i.e. splines and NURBS, to also represent PDE unknowns. The smoothness of splines is a new ingredient that yields several advantages: for example, it improves the accuracy per degree of freedom and allows for the direct approximation of higher order PDEs. Isogeometric methods have been used and tested on a variety of problems of engineering interests, for flow simulations [2–9], and for electromagnetic problems [10–12].

In electromagnetic and incompressible fluids flow simulations, numerical discretizations have to preserve the geometric structure of underlying PDEs in order to avoid spurious behaviors, instability or non-physical solution. Thus the numerical discretizations have to be related through a discrete De Rham complex. Compatible spaces for finite element approximations were in general introduced by Arnold et al. [13], and more recently in isogeometric analysis context,

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http://dx.doi.org/10.1016/j.cma.2015.03.028
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by Buffa et al. [5,14]. High regularity of splines is advantageous for constructing compatible spaces. Some initial work to show the potential impact of compatible spline based methodology for electromagnetic wave computations was presented in [10,12] and recently using T-spline complexes in [11]. The first isogeometric discretizations of incompressible fluid flows was done by Bazilevs et al. [15], generalizing Taylor–Hood elements to NURBS. Later, Buffa et al. [5] investigated a solver based on three choices of discrete spline spaces to approximate the mixed discretization for Stokes problem, which were seen as a smooth extension of Taylor–Hood (TH), Nedelec (N) and Raviart–Thomas (RT) pair of Finite Element (FE) spaces. One of their main finding was the smooth RT pair of spline based FE spaces provides divergence-free discrete solutions. Later using the idea of div-compatible spline spaces presented in the setting of discrete differential forms [14], a series of isogeometric divergence conforming spline discretizations were derived to solve Stokes and Brinkman equations, steady and unsteady Navier–Stokes equations by Evans in [8,9,16]. These initial developments show that isogeometric analysis is a highly accurate and efficient methodology to solve incompressible flow problems. In this paper our aim is to extend the div-compatible spaces to locally refined meshes and explore the benefit of adaptive refinement in solving incompressible flow problems. Adaptivity and local refinement allows us to not only achieve optimal convergence by resolving strong singularities, but also allows us to resolve local behavior such as recirculation eddies in fluid flow. While the framework presented here is formulated on an unmapped rectangular domain, it is conjectured that it is possible to extend it to mapped geometries using the Piola mapping in the same way as [5].

Non-uniform rational B-splines (NURBS) is the dominant geometric representation format for CAD. The construction of NURBS is based on a tensor product structure and, as a consequence, knot insertion is a global operation. To remedy this a local refinement can be achieved by breaking the global tensor product structure of multivariate splines and NURBS. Several techniques have been proposed to address this, among others are T-splines [17,18], Hierarchical B-splines [19,20], Truncated Hierarchical B-splines [21] and Locally Refined (LR) B-splines [22]. While initially, most of the references address the problem from a CAD point of view, later years have seen them applied to isogeometric analysis. For T-splines consider [23–27], for Hierarchical B-splines consider [28–31], and for LR-splines see [32,33].

1.1. Aim and outline of the paper

The aim of this paper is to present a class of compatible spline spaces with local refinement capability which form a De Rham complex and provide a stable, divergence-free discretization of the 2D Stokes problem.

The paper is organized as follows:

We start in Section 2 with our model Stokes problem which can be seen as a prototype of viscous incompressible flows. The necessary conditions to derive a divergence conforming spline discretization and the main results of the paper are presented.

In Section 3, we introduce the basic concepts of splines over locally refined Box-meshes (or T-meshes). We present the dimension formula as given by Mourrain [34] which will be useful in proving the compatibility among the splines spaces on locally refined meshes. Further we discuss the construction of derivatives spaces on locally refined meshes.

In Section 4, we present three different complete De Rham complexes on locally refined meshes. The complexes are characterized by their boundary conditions for the velocity space: (i) without boundary conditions, (ii) with no penetration boundary conditions, and (iii) with no slip boundary conditions.

To build a basis on the Box meshes we introduce the locally refined (LR) B-splines in Section 5. We give a brief overview of their generality before defining a subclass which we will use for the local refinement. This is the structured mesh refinement as introduced in [32].

In Section 6, we present some numerical results for Stokes problems. The numerical stability, convergence rates, efficiency and conservation properties of the proposed LR B-spline discretization in adaptive isogeometric analysis will be our main focus.

Finally, some conclusions and perspectives are included in Section 7.

2. Stokes problem and divergence-conforming spline discretization

To model a viscous incompressible flow, we consider the following Stokes problem: find $(\mathbf{u}, p)$ such that

$$\begin{cases} -\nu\nabla^2\mathbf{u} + \nabla p = \mathbf{f} & \text{in } \Omega \\ \nabla\cdot\mathbf{u} = 0 & \text{in } \Omega \end{cases} \tag{1}$$

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