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Polyhedral elements with strain smoothing for coupling hexahedral meshes at arbitrary nonmatching interfaces

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Abstract

Managing nonmatching interfaces between dissimilar hexahedral meshes is complicated because the interfaces, which are arbitrarily formed by the intersection of element-edges, are composed of various types of polygons. In this paper, a simple and efficient scheme is proposed for dealing with nonmatching interfaces using polyhedral elements. Because polyhedral elements can have arbitrary numbers of polygonal faces and nodes, they can be used as transition elements for coupling nonmatching meshes. A strain smoothing technique in the cell-based smoothed finite element method is introduced to resolve critical difficulties in defining shape functions and in conducting numerical integration for the polyhedral elements, as well as to further enhance the accuracy and efficiency of finite element analyses. The effectiveness of the proposed scheme is demonstrated through several numerical examples involving nonmatching interfaces. The effects of decomposition types of smoothing cells in the strain smoothing technique are also discussed in terms of numerical stability and accuracy.

Keywords: Polyhedral element; Strain smoothing; Cell-based smoothed finite element method; Transition element; Nonmatching meshes

1. Introduction

Nonmatching interfaces commonly occur in finite element (FE) analyses of various problems. For example, when a complex structure is spatially discretized, decomposing itself into several substructures [1,2], nodes on interfaces between the independently modeled substructures may not be shared with dissimilar meshes for the substructures. That is, conditions of nodal connectivity and element compatibility are not satisfied at the nonmatching interfaces. Adaptive mesh refinement [3–6], multiscale simulations [7–9], and multiphysics simulations [10–12] can also involve nonmatching interface problems. If such nonmatching interface problems are not resolved in an appropriate manner, the convergence and accuracy of solutions obtained by the finite element method (FEM) cannot be guaranteed.

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Because of advances in mesh generation schemes, meshes at nonmatching interfaces can be modified or regenerated [13–15]; however this not only involves a cumbersome process, but it also offsets an advantage of independently modeled meshes. As a result, a variety of schemes have been proposed to manage nonmatching interfaces while maintaining individual meshes. There are methods that impose constraint conditions for matching nodal behaviors at nonmatching interfaces between dissimilar meshes: Lagrange multipliers [1,16–19], and penalty function parameters [2,20] have been mainly used for applying the constraints. These methods, however, require modifications of a global stiffness matrix, and thus it may become nonpositive-definite and ill-conditioned. In contrast, there are methods that use nonconventional types of elements for accommodating arbitrary distributions of nodes on nonmatching interfaces. To name but a few, interface elements [21–23] and variable-node elements [3–8,24–28] have been used as transition elements for gluing dissimilar meshes. The use of these elements enables a global stiffness matrix to be assembled in a straightforward way, without modifications.

This paper is concerned with the coupling of three-dimensional nonmatching meshes using nonconventional types of elements. In particular, a simple and efficient scheme is proposed to manage arbitrarily shaped nonmatching interfaces resulting from dissimilar hexahedral meshes. Variable-node elements have been proposed and applied to mesh coupling, which can contain arbitrary numbers of additional nodes on a four-node quadrilateral element in two dimensions [5–8,24–26], and on an eight-node hexahedral element in three dimensions [3–5,27,28]. However, with three-dimensional mesh coupling, variable-node elements are not generally applicable for arbitrary nonmatching interfaces. This is because they have thus far allowed only regular distributions of additional nodes in three dimensions [3–5]. Recently, Sohn et al. [28] further enhanced the flexibility of nodal distributions of three-dimensional variable-node elements; nonetheless, types of the nodal distributions have not been generalized completely. Meanwhile, Kim [23] proposed three-dimensional interface elements and achieved a hexahedral mesh connection at arbitrary nonmatching interfaces; however, there is still room for improvement in terms of accuracy and efficiency, because shape functions of the interface elements were formulated by moving least square approximation.

In this paper, polyhedral elements are used to achieve the successful coupling of dissimilar hexahedral meshes involving arbitrary nonmatching interfaces, in an efficient manner. Based on the configurations of the individual meshes, nonmatching interfaces are formed in combinations of arbitrary polygons. Consequently, polyhedral elements with arbitrary polygon-shaped faces can be used as transition elements between general nonmatching meshes. That is, hexahedral elements on the nonmatching interfaces are replaced by polyhedral elements, and thus compatible meshes are constructed without any gaps or overlaps.

However, critical issues involving how to define shape functions and how to conduct numerical integration for polyhedral elements still remain. In most studies on polyhedral elements for use in FE analyses [29–34], these issues have been resolved through difficult and complicated processes. Here, to facilitate an efficient treatment of the shape functions, a strain smoothing technique in the cell-based smoothed finite element method (CSFEM) [8,35–44] is introduced. Since the CSFEM was proposed by Liu et al. [35], it has been applied to various mechanics problems as a substitute for the conventional FEM, due to its excellent features [36–40]. Recently, Sohn et al. [44] presented polyhedral elements with strain smoothing, however, this previous study focused on applying polyhedral elements for efficient mesh generation, rather than investigating the effects of smoothing cells to element properties. Although there are no universal rules to construct smoothing cells in the CSFEM, solution behaviors are affected by the number and shape of the smoothing cells. The present paper systematically discusses numerical stability through an eigenvalue analysis according to decomposition types of smoothing cells, and further demonstrates that polyhedral elements are effective for a seamless connection of nonmatching meshes.

The remainder of this paper is organized as follows. Section 2 describes the formation process of nonmatching meshes, and presents an efficient and direct scheme to connect dissimilar hexahedral meshes using polyhedral elements. Section 3 discusses how to deal with polyhedral elements in the framework of the CSFEM. In particular, four types of decomposing an element into smoothing cells are considered. Section 4 presents numerical examples involving arbitrary nonmatching interfaces in order to demonstrate the effectiveness of the proposed scheme. Finally, Section 5 provides some concluding remarks.

2. Connection of arbitrary nonmatching meshes using polyhedral elements

Nonmatching interfaces in two dimensions may appear in the form of lines because they are formed between element-edges of dissimilar meshes, whereas interfaces in three dimensions may be constructed in the form of surfaces