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Isogeometric Kirchhoff-Love shell formulations for biological membranes

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Abstract

Computational modeling of thin biological membranes can aid the design of better medical devices. Remarkable biological membranes include skin, alveoli, blood vessels, and heart valves. Isogeometric analysis is ideally suited for biological membranes since it inherently satisfies the C^1-requirement for Kirchhoff-Love kinematics. Yet, current isogeometric shell formulations are mainly focused on linear isotropic materials, while biological tissues are characterized by a nonlinear anisotropic stress-strain response. Here we present a thin shell formulation for thin biological membranes. We derive the equilibrium equations using curvilinear convective coordinates on NURBS tensor product surface patches. We linearize the weak form of the generic linear momentum balance without a particular choice of a constitutive law. We then incorporate the constitutive equations that have been designed specifically for collagenous tissues. We explore three common anisotropic material models: Mooney-Rivlin, May Newmann-Yin, and Gasser-Ogden-Holzapfel. Our work will allow scientists in biomechanics and mechanobiology to adopt the constitutive equations that have been developed for solid three-dimensional soft tissues within the framework of isogeometric thin shell analysis.

Keywords: isogeometric analysis, thin shells, Kirchhoff-Love kinematics, biological membranes

1. Motivation

Biological membranes appear often in nature, fulfilling crucial physiological roles for the survival of different forms of life. Perhaps one of the most evident examples is our skin, an essential barrier from the outside world with notable elastic properties [52]. Several other examples of membranes - although hidden to our eyes - are equally important because of their prominent functions: the alveoli, the pericardium, or the valve leaflets, to name just a few [41]. Characterizing the behavior of these thin structures in distinct mechanical scenarios is key to improve our understanding of the mechanical aspects of disease and to design more effective medical devices [34]. Biological membranes are lightweight structures that often experience large deformations, large rotations, and extreme membrane strains [43]. The mechanical behavior of most biological membranes is a result of the well-defined tissue microstructure: a water-based matrix, often considered incompressible, in which fibers such as collagen form a complex network responsible for tissue anisotropy and nonlinearity [7].

Thin membranes can represented using Kirchhoff-Love kinematics. This strategy has been deemed appropriate and verified with experiments for thin biological structures including skin and heart leaflets, which show thicknesses that rarely exceed a few millimeters [15, 48]. While the physiological loading situation is often associated with a plain membrane state, where no bending energy is considered [30], many applications of interest deal with diseased and non-physiological scenarios for which bending stresses may become critical [37]. This thin shell approach requires a high continuity representation of the domain, which has traditionally been an obstacle for conventional finite element implementations. Recently, however, the development of isogeometric analysis tools has made it possible to develop Kirchhoff-Love shells that easily satisfy the requirement of C^1 continuity across element boundaries [31]. Yet, to date, mainly linear St. Venant-Kirchhoff materials have been used within this approach [25]. While the St. Venant-Kirchhoff model provides reasonable results in the large-deformation-small-strain regime, it might be inappropriate for biological tissues, which are typically anisotropic, nonlinear, and subjected to large strains [11].

Here we present a isogeometric shell formulation especially tailored for thin biological membranes. We employ the Kirchhoff-Love kinematic assumption and represent the geometry of a three-dimensional elastic body by parametrizing the mid surface using tensor product NURBS surface patches. We present the standard virtual work formulation of the equations of mechanical equilibrium and perform the generic consistent linearization without choosing a particular constitutive model. We then explore the constitutive equations available for biological membranes and incorporate them into our formulation by imposing the plane stress assumption. Finally, we demonstrate the performance of the formulation by selected numerical examples.

The formulation presented here departs from currently available isogeometric shell models in that our virtual work and consistent linearization are expressed for general constitutive models in the neighborhood of the mid-surface by avoiding the explicit integration across the thickness. This flexibility allows us to ex-

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