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Improved robustness for nearly-incompressible large deformation meshfree simulations on Delaunay tessellations

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Abstract

A displacement-based Galerkin meshfree method for large deformation analysis of nearly-incompressible elastic solids is presented. Nodal discretization of the domain is defined by a Delaunay tessellation (three-node triangles and four-node tetrahedra), which is used to form the meshfree basis functions and to numerically integrate the weak form integrals. In the proposed approach for nearly-incompressible solids, a volume-averaged nodal projection operator is constructed to average the dilatational constraint at a node from the displacement field of surrounding nodes. The nodal dilatational constraint is then projected onto the linear approximation space. The displacement field is constructed on the linear space and enriched with *bubble-like* meshfree basis functions for stability. The new procedure leads to a displacement-based formulation that is similar to F-bar methodologies in finite elements and isogeometric analysis. We adopt maximum-entropy meshfree basis functions, and the performance of the meshfree method is demonstrated on benchmark problems using structured and unstructured background meshes in two and three dimensions. The nonlinear simulations reveal that the proposed methodology provides improved robustness for nearly-incompressible large deformation analysis on Delaunay meshes.

Keywords: Hyperelasticity; Large deformations; Meshfree methods; Maximum-entropy approximation; F-bar method; Delaunay meshes

1. Introduction

In nearly-incompressible analysis of solids that undergo large deformations, mesh distortion introduces a limitation for practical use of simplicial (Delaunay) tessellations within the framework of standard finite elements. Three-node triangular and four-node tetrahedral finite elements are not used for nearly-incompressible analysis of solids because they lead to volumetric locking. However, they can be suitably modified for nearly-incompressible settings through the displacement/pressure mixed formulation (u–p form). The realization of these finite elements is the well-known `MINI` element [1], where the nodes located at the vertices of the simplicial element are used to interpolate continuous

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linear displacement and continuous linear pressure fields. In addition, the displacement field is enriched with an interior node located at the barycenter of the simplicial element. This extra node is related to a cubic (bubble) basis function that vanishes on the element boundary and renders an inf–sup stable element [2–4]. Although the MINI element demonstrated better stability properties than several finite element formulations in certain finite deformation regimes [5], the shape function's dependence on the Delaunay tessellation makes it very sensitive to mesh distortion. In this paper, a new methodology on Delaunay meshes is proposed for the meshfree analysis of nearly-incompressible solids at finite strains that is superior to the MINI element formulation.

In the literature, the poor performance of simplicial tessellations in large deformation analysis of nearly-incompressible solids has been improved through various techniques such as mixed-enhanced elements [6–8], pressure stabilization [9–11], composite pressure fields [12–14], and average nodal pressure/strains [15–20]. The last two approaches are broadly based on the idea of reducing pressure (dilatational) constraints to alleviate volumetric locking. In meshfree methods, nodal integration techniques [21] can be considered to be indirectly related to methods that use simplicial tessellations since their formulation is based on the dual of the Delaunay triangulation, that is, the Voronoi diagram. In this approach, the fewer constraints that are met by performing numerical integration only at the nodes permits to alleviate volumetric locking. However, the drawback of nodal integration techniques is their instability, which has motivated studies to stabilize them [22,23].

In contrast to finite elements, meshfree methods are constructed using basis functions that possess larger supports and do not rely on a mesh for their definition. This allows meshfree methods some degree of insensitivity to mesh distortions, thus providing us with the motivation to use meshfree basis functions in this paper. Nonetheless, a background mesh is still required in Galerkin meshfree methods to perform the numerical integration of the weak form integrals. In the meshfree method that is developed herein, background meshes of three-node triangles in two dimensions and four-node tetrahedra in three dimensions, are used.

Volumetric locking remains an issue in meshfree methods that use simplicial tessellations for numerical integration in nearly-incompressible media problems. Thus, a special procedure needs to be developed to alleviate volumetric locking. To this end the nonlinear version of the *volume-averaged nodal projection* method (referred to as VANP in Ref. [24]) proposed for small strain elasticity in Ref. [25] is developed to average the dilatational constraint at a node from the displacement field of surrounding nodes. The nodal dilatational constraint is then projected onto the linear approximation space. The displacement field is constructed on the linear space and enriched with *bubble-like* meshfree basis functions for stability. The formulation so devised leads to a displacement-based method that shares some common features with the F-bar-Patch method of Ref. [26] and the isogeometric F-bar projection method of Ref. [27], and as such, it can be regarded as an F-bar methodology for meshfree methods. In the numerical implementation, maximum-entropy basis functions are used as the meshfree basis functions. Another approach that uses bubble functions to address volumetric locking for low-order simplicial tessellations is proposed in Ref. [28] for compressible and nearly-incompressible linear elastic solids and in Refs. [29,30] for large deformations. Wu and Koishi [30] use the conforming nodal integration procedure of Chen et al. [31] to suppress locking, whereas in our approach, the locking-free behavior stems from a u–p mixed formulation in which a volume-averaged technique is used to eliminate the pressure degrees of freedom from the analysis. Furthermore, the smoothing in Ref. [30] is done over the covering that is formed by the bubble nodes that are neighbors to an element face and the nodes that define that face, whereas in this work the volume-averaging is done over the region of support of the vertex basis functions.

In a Galerkin-based meshfree method, the integration domain is a cell that typically does not coincide with the region that is defined by the intersecting supports of two overlapping meshfree basis functions. In addition, meshfree basis functions are rational (nonpolynomial) functions. These are two central issues that introduce numerical errors when using standard Gauss quadrature for numerical integration. The errors can be reduced by using a large number of Gauss points per cell; however, this substantially increases the computational costs in the numerical integration. There have been many attempts to correct these integration errors. An early contribution was due to Dolbow and Belytschko [32], who proposed to use integration cells that were aligned with the support of the nodal basis functions. Since then, many other approaches have been pursued (for instance, see Refs. [33,34]). Babuška and coworkers have provided the theoretical basis for the numerical integration issue in first-order meshfree methods [35] as well as higher-order meshfree approximations [36]. Other approaches that are based on nodal integration ideas [31] construct a strain correction that significantly reduces integration errors. Ortiz et al. [25] proposed a strain correction based on a smoothing procedure for linear approximations on triangular and quadrilateral background meshes and extended these ideas to tetrahedral background meshes in Ref. [37]. Duan et al. [38] proposed a smoothing procedure for