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A coupling technique for non-matching finite element meshes

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Abstract

This paper presents a novel technique for coupling non-matching finite element meshes, based on the use of special finite elements termed *coupling finite elements* (CFEs), which share nodes with non-matching meshes. The main features of the proposed technique are: (i) no additional degree of freedom is introduced to the problem; (ii) non-rigid coupling can be considered to describe the nonlinear behavior of interfaces similar to cohesive models; (iii) non-matching meshes of any dimension and any type of finite elements can be coupled, and (iv) overlapping and non-overlapping meshes can be considered. The applicability of the proposed technique is illustrated by a variety of 2D and 3D examples with different non-matching mesh configurations. The results demonstrate that the technique is able to enforce the continuity of displacements in the case of rigid coupling, and to properly transfer the interaction forces across the non-matching interfaces, according to any chosen interface model, in the case of non-rigid coupling.

Keywords: Finite elements; Non-matching meshes; Coupling scheme; Interface model; Multiscale analysis

1. Introduction

The accuracy of results in numerical analysis by the finite element method is related directly to the adequacy of the discretization. However, the finer the mesh the greater the computational effort required to solve the problem. Thus, a common solution for large scale problems is to use a fine mesh only in the region of interest. As a consequence, another problem may arise in the transition between coarse and fine meshes, since the presence of distorted elements can invalidate the solution in the transition regions [1]. Another strategy widely used today consists of discretizing the regions of the problem (subdomains), in a totally independent way, according to the interest of the analyst, and then use a coupling technique to connect their non-matching interfaces. This strategy has been applied extensively to problems with adaptive mesh refinement [2–4], multiscale problems [5–7] or multiphysics analysis [8–11]. In addition, with

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the advent of parallel computing, this kind of approach has been extensively used to deal with the interaction effects between the subdomains, initially subdivided to be computed by different processors [12,13].

In this context, a number of coupling methods have been developed to capture interface effects accurately [14]. Ref. [15] defines the continuity and compatibility conditions at non-matching interfaces between subdomains as the fundamental requirements, and, also assert that the strain fields must be transferred correctly through the non-matching interfaces. Constraint equations have been used to make a strong coupling between the subdomains. This method is usually known as multi-point constraint (MPC). The main idea is to evaluate the displacement of the loose coupling boundary nodes of the local subdomain (fine mesh) using the displacement interpolation of the adjacent finite element of the global subdomain (coarse mesh) [5].

Other existing classes of methods for coupling non-matching meshes are based on a weak coupling approach. In this case, the displacement compatibility is only satisfied in an average sense. Consequently, displacement compatibility between the subdomains may not be satisfied, which may result in small gaps or overlaps [5].

Another general classification for the methods to couple non-matching meshes is to divide them as dual or primal methods. In the dual approaches [14] the methods are based on the use of a field of Lagrange multipliers to impose the boundary conditions, as for instance, in the mortar method [16,17] and the Arlequin method [18,19]. Unlike the former, the Arlequin method presents a region in which the subdomains overlap. The main drawback of these methods is the introduction of extra unknowns (additional degrees of freedom) to the system of equations.

On the side of the primal methods one could cite the penalty methods [20,21]. Also the discontinuous Galerkin (DG) methods (a good review of DG methods is given in [22]) and Nitsche methods (originally described in [23]) are among the most widely used. In these methods, the interface is represented by its displacement field and no dual variables are introduced [14]. For this reason, and in contrast with the dual approaches, primal methods are not subject to the *inf–sup* or Ladyzhenskaya–Babuška–Brezzi (LBB) restrictions. However, a stabilization parameter is needed. Ref. [14] proposed a primal interface formulation that uses local enrichment of the interface elements to enable an unbiased enforcement of geometric compatibility at all interface nodes without inducing over-constraint and additional variables. In [24] some primal coupling methods such as the nearest neighbor interpolation [25], projection method [26] and methods based on spline interpolation [25,27] are compared for problems of fluid–structure interaction.

Another class of coupling strategies are the methods that convert standard finite elements on non-matching interfaces into special types of elements. Methods based on the interface element method (IEM) [28,15] may be included in this class. In these methods, interface elements are defined on the finite elements bordering the non-matching interfaces, and the moving least square (MLS) approximations are used to construct the shape functions of these interface elements [15]. This approach introduces no additional degrees of freedom, and hence the system matrix remains positive-definite. However, the algorithm for the construction of interface elements is not negligible and is computationally expensive. Moreover, it is necessary to store information about the shape functions at all the integration points in the interface region, according to the conventional way to calculate shape functions using MLS [29]. Based on the IEM, [30] developed three-dimensional interface elements for coupling non-matching hexahedral meshes. To overcome the main disadvantages of the IEM, [29] proposed a new improved interface element called MLS-based variable-node element. Other applications using this strategy can be found in [31–34]. In the literature some authors call the interface elements transition elements [3,35].

This paper presents a novel approach for coupling non-matching meshes based on the use of the *coupling finite elements* (CFEs). As a primal approach it has the advantage that no additional degrees of freedom are added to the system of equations. Moreover, as the CFEs use the same shape functions of the standard finite elements of the reference mesh, special formulation or integration rule for enriched elements are not required, as in the case of variable-node interface elements (see, e.g., [30]). However, as a penalty method, the proposed approach may cause ill-conditioning of the stiffness matrix as a drawback which can be overcome by the use of suitable values for the penalty variables.

As will be shown, the inclusion of an adequate number of CFEs in the mesh can ensure the continuity of displacement between originally independent meshes without increasing the total number of degrees of freedom of the problem. Moreover, this new approach can be also applied to overlapping meshes. In this context, the formulation of the strategy is extended to couple linear reinforcement elements embedded in 2D and 3D meshes of the matrix. For this case, a continuum damage model is used to describe the bond–slip behavior between the matrix and reinforcement.

The remainder of this paper is organized as follows. Section 2 describes the idea behind the strategy developed for coupling non-matching meshes. The CFE formulation is detailed in Section 3. Modeling of rigid coupling (perfect