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Half-explicit timestepping schemes on velocity level based on time-discontinuous Galerkin methods

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Highlights

- We propose a time-integration method for dynamic contact problems with rigid and flexible bodies.
- We deal consistently and automatically with friction and impulses if necessary.
- We offer automatic order elevation if possible.
- We provide an efficient half-explicit method on velocity level with reduced drift-off effect.
- We demonstrate validity and computing time reduction for various numerical examples.

Abstract

This paper presents a time-discretization scheme for the simulation of nonsmooth mechanical systems. These consist of rigid and flexible bodies, joints as well as contacts and impacts with dry friction. The benefit of the proposed formalism is both the consistent treatment of velocity jumps, e.g. due to impacts, and the automatic local order elevation in non-impulsive intervals at the same time. For an appropriate treatment of constraints in impulsive and non-impulsive intervals, constraints are implicitly formulated on velocity level in terms of an augmented Lagrangian technique (Alart and Curnier, 1991). They are satisfied exactly without any penetration. For efficiency reasons, all other evaluations are explicit which yields a half-explicit method (Brasey, 1994a,b; Murua, 1995, 1997; Arnold et al., 1998; Hairer et al., 2009, 2010).

The numerical scheme is an extended timestepping scheme for nonsmooth dynamics according to Moreau (1999). It is based on time-discontinuous Galerkin methods to carry over higher order trial functions of event-driven integration schemes to consistent timestepping schemes for nonsmooth dynamical systems with friction and impacts. Splitting separates the portion of impulsive contact forces from the portion of non-impulsive contact forces. Impacts are included within the discontinuity of the piecewise continuous trial functions, i.e., with first-order accuracy. Non-impulsive contact forces are integrated with respect to the local order of the trial functions. In order to satisfy the constraints, a set of nonsmooth equations has to be solved in each time step depending on the number of stages; the solution of the velocity jump together with the corresponding impulse yields another nonsmooth equation. All nonsmooth equations are treated separately by semi-smooth Newton methods.

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The integration scheme on acceleration level was first introduced in Schindler et al. (2013) labeled "forecasting trapezoidal rule". It was analyzed and applied to a decoupled bouncing ball example concerning principal suitability without taking friction into account. In this work, the approach is algorithmically specified, improved and applied to nonlinear multi-contact examples with friction. It is compared to other numerical schemes and it is shown that the newly proposed integration scheme yields a unified behavior for the description of contact mechanical problems.

Keywords: Timestepping scheme; Discontinuous Galerkin method; Nonsmooth dynamics; Flexible multibody system; Impact; Index reduction

1. Introduction

In this paper, we study numerical integration schemes for the simulation of nonsmooth mechanical systems. Rigid and flexible bodies, joints as well as contacts and impacts with dry friction constitute the mechanical models. Thereby, we formulate the contact conditions as constraints and do not allow any penetration, e.g. due to penalty techniques. As a result, velocity jumps occur during the transient simulation of semi-discrete models and we have to be cautious in the formulation of efficient and stable time-discretization schemes.

We distinguish two cases.

1. *Non-impulsive contact forces* — For the contact between flexible bodies, the contact force is finite in continuum models, i.e., in not semi-discrete models, although velocity jumps may arise. Classic (implicit) time integration schemes for computational mechanical problems, i.e., members of the Newmark family [1–5], have been adapted to these demands and extended with respect to contact/velocity updates (Laursen–Love scheme) [6, 7]. Another strategy to preferably get a well-posed problem is the application of energy–momentum paradigms like in energy–momentum schemes [8,5], i.e., modifications of the midpoint rule [4], to impact problems (Laursen–Chawla scheme) [6]. A contact-stabilized Newmark scheme is proposed in [9].
2. *Impulsive contact forces* — For the contact between rigid bodies, the reaction forces are impulsive and the classic time integration schemes do not work anymore [10]. The application of mass redistribution techniques [7] is a procedure, which reminds of penalty approaches with the benefit of having a theoretical foundation [11]. However in [12], it is shown that all these schemes suffer from oscillations in the relative contact velocities. Event-driven schemes and timestepping schemes are further concepts to simulate rigid multibody systems or semi-discrete systems consistently by applying impact laws. Thereby, event-driven schemes resolve impact events to a high precision. In-between impact events, standard integration schemes are used. Classic timestepping schemes do not resolve impact events, but include their possible existence directly in the discretization. Thus, they have low accuracy in non-impulsive intervals, whereas event-driven schemes may get inefficient and inconsistent for many impact events [10].

The aim of the present paper is to improve the consistent and robust concept of timestepping schemes for semi-discrete mechanical systems. The main drawback is the lack of problem adaptive accuracy in non-impulsive intervals. Classic timestepping schemes can be embedded within the context of time-discontinuous Galerkin methods, when we choose piecewise constant trial functions for the velocity approximation [13]. In the aforementioned paper, two different families of timestepping schemes on acceleration level based on discontinuous Galerkin methods have been introduced and analyzed. They differ in the interpretation, if one assumes the velocity jump at the beginning, called D^+ timestepping schemes, or at the end, called D^- timestepping schemes, of each discretization interval. First, using higher-order but piecewise continuous trial functions for the velocity, and, second, splitting of impulsive and non-impulsive contact reactions, offer the opportunity to both stay consistent and benefit from a higher-order integration of non-impulsive contact reactions. The "forecasting trapezoidal rule" is the D^- representative for piecewise linear velocity trial functions; concerning function evaluations and implementation complexity, it is the easiest scheme. Its basic practical applicability apart from theoretical propositions has been shown with a decoupled bouncing ball example. It is the basis for extensions in the present paper. We algorithmically specify and improve it and apply it to nonlinear multi-contact examples with friction. Thereby for an automatic switching between non-impulsive and impulsive intervals, constraint equations for non-impulsive reactions are also formulated on velocity level in an augmented Lagrangian setting [14] like the constraint equations for contact impulse and jumping velocity. Therefore,