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Polygon-based contact description for modeling arbitrary polyhedra in the Discrete Element Method

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Abstract

Many real-life applications of the Discrete Element Method (DEM) require a particle description which accounts for irregular and arbitrary shapes. In this work, a novel method is presented for calculating contact force interactions between polyhedral particles. A contact between two polyhedra is decomposed as a set of contacts between individual polygonal facets. For each polygon–polygon contact, an individual contact force is obtained by integrating a linear pressure over the area of its intersection. Both convex as well as partially concave polyhedra can be accurately represented. The proposed algorithm is validated by comparing to previously published experimental and computational gravitational particle depositions of identical cubes. Finally, the model is demonstrated in simulations of gravitational packing of various other polyhedral shapes.

Keywords: Discrete Element Method; Arbitrary shapes; Contact model; Polyhedra; Gravitational packing

1. Introduction

In granular assemblies, particle shape has been shown to be a determining parameter affecting, among else, a system's response upon loading [1,2], packing density, stress patterns [3] and ratcheting behavior [4]. In the Discrete Element Method (DEM), which tries to describe granular systems as assemblies of distinct, explicitly modeled bodies interacting by means of contact forces [5], particle shape is often approximated using a simplified geometrical representation, e.g. spheres. Many applications, however, require a more elaborate description of irregular bodies.

During the last years, many advances are made in shape description for the Discrete Element Method. Instead of spheres, ellipsoids [6–8], superquadrics [9,10], and polyhedra [5,11] have been used to approximate particle shape. Other approaches use composites of more simple shape primitives, such as spheres [12–14], ellipsoids [15] and spheropolygons [16]. A variation of DEM, the Granular Element Method (GEM), uses Non-Uniform Rational Basis-Splines (NURBS) to capture grain shape, offering a flexible and robust algorithm to account for arbitrary rounded

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shapes [17,18]. Another method for modeling arbitrary rounded shapes is based on triangulated surface meshes in which the local curvature is used for a Hertzian contact force formulation [19].

In this work, we propose a flexible and easy-to-implement algorithm to model irregular polyhedral particles. The presented method represents particles using a surface mesh containing polygonal facets, and formulates contact forces based on individual interactions between two contacting bodies. For each polygon–polygon contact, a linear elastic and dissipative pressure is used which is numerically integrated over the intersection of the two polygons. Because each contact between two polygons is resolved independently, the method benefits from efficient contact detection and can be easily parallelized. In Section 2, it is explained how contact forces can be computed between two arbitrarily shaped polyhedra. Next, in Section 3, the model is validated by comparing to simulations of gravitational deposition of cubes and further demonstrated by showing analogous deposition of various other polyhedral particles.

2. Model description

2.1. Contact detection

Contact detection, i.e. the generation of a list of contact candidates, is performed on the level of individual polygonal facets, instead of between two complete polyhedral bodies. Bounding boxes [20] are constructed for each individual polygon. Using these bounding boxes several efficient contact detection methods can be applied, such as (multi-)grid [21–24] and octree [25,26] methods.

For each set of two polygons, these algorithms can cheaply determine whether or not their bounding boxes are overlapping, and are therefore likely to have physical contact. With these contact detection methods, the computational effort does not scale with the number of polygons being used in the simulation, but only with the number of polygons that are actually in contact (see Section 3.4 and [19]).

2.2. Geometrical contact properties

Contact pressures are calculated on the contact plane between two polygons P_1 and P_2 with normal vectors $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$. For this, an intersection polygon S_{12} is first determined. In the case of equal material properties, the plane in which S_{12} lies is chosen as the bisection of the planes of P_1 and P_2. The contact normal unit vector is therefore approximated as:

$$\hat{\mathbf{n}}_{12} = \frac{\hat{\mathbf{n}}_2 - \hat{\mathbf{n}}_1}{\left\| \hat{\mathbf{n}}_2 - \hat{\mathbf{n}}_1 \right\|}. \tag{1}$$

If the two contacting bodies have a different stiffness, the contributions of $\hat{\mathbf{n}}_1$ and $\hat{\mathbf{n}}_2$ to $\hat{\mathbf{n}}_{12}$ should in principle be inversely weighted with their stiffness.

All three planes characterized by $\hat{\mathbf{n}}_1$, $\hat{\mathbf{n}}_2$ and $\hat{\mathbf{n}}_{12}$ contain the plane–plane intersection line defined by the vector $\hat{\mathbf{l}}_{12} = \hat{\mathbf{n}}_1 \times \hat{\mathbf{n}}_2$ and a point $\mathbf{s}$ chosen on the intersection line.

Next, P_1 and P_2 are projected on the contact plane along the direction of respectively $\hat{\mathbf{n}}_2$ and $\hat{\mathbf{n}}_1$, yielding the projections P_1' and P_2' (see Fig. 1(a) and (c)). S_{12} is then obtained by computing the side of the intersection between P_1' and P_2' which is in the direction of positive overlap (Fig. 1(b)).

At a given test point $\mathbf{x}$ inside S_{12}, the overlap distance δ_{12} can be calculated as:

$$\delta_{12}(\mathbf{x}) = 2\tan(\alpha)\left[(\mathbf{x} - \mathbf{s}) \cdot (\hat{\mathbf{n}}_{12} \times \hat{\mathbf{l}}_{12})\right] \tag{2}$$

with $\cos(\alpha) = \hat{\mathbf{n}}_{12} \cdot \hat{\mathbf{n}}_1$. The contact point $\mathbf{c}$ is approximated as the mean of the corners of S_{12}, weighted by their corresponding overlap distance according to Eq. (2).

In every $\mathbf{x} \in S_{12}$, a relative contact velocity is defined as:

$$\mathbf{v}_{12}(\mathbf{x}) = \mathbf{v}_2^{\mathrm{dof}} - \mathbf{v}_1^{\mathrm{dof}} + \mathbf{w}_2^{\mathrm{dof}} \times (\mathbf{x} - \mathbf{x}_2^{\mathrm{dof}}) - \mathbf{w}_1^{\mathrm{dof}} \times (\mathbf{x} - \mathbf{x}_1^{\mathrm{dof}}), \tag{3}$$

where $\mathbf{x}_i^{\mathrm{dof}}$, $\mathbf{v}_i^{\mathrm{dof}}$ and $\mathbf{w}_i^{\mathrm{dof}}$ are respectively the center of mass position, velocity and angular velocity of the polyhedron to which polygon P_i belongs.

To deal with issues of numerical accuracy – e.g. exact flat contacts – or efficiency – e.g. early contact reject cases, additional calculations are performed. These are briefly summarized in Appendix A.