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# A Generalized Finite Element Method for hydro-mechanically coupled analysis of hydraulic fracturing problems using space-time variant enrichment functions

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## Highlights

- A novel Generalized Finite Element Method (*GFEM*) for hydraulic fracturing problems is proposed.
- Physically motivated enrichment functions are used to approximate the liquid pressure near cracks.
- The enrichment functions are adopted from the analytical solution of 1D consolidation type problems.
- The enrichment functions are space and time variant.
- A considerable reduction of the approximation error compared to existing *FEM* and *XFEM* models is obtained.

## Abstract

In computational simulations of hydraulic fracturing problems, consideration of interactions between the propagating fracture zone and the fluid flow through the porous material requires an appropriate up-scaling procedure from the spatial scale of the local crack, which usually is much smaller compared to the scale of typical finite elements in poromechanics problems. This scale transition refers to both the displacement field (discontinuity across cracks) as well as to the fluid flow (accelerated flow within cracks and the interaction with the flow in the bulk material). To resolve the small and the large scale portion of the solution, the Generalized Finite Element Method (*GFEM*) exploiting the partition of unity property of shape functions is used. Accordingly, the displacements $\mathbf{u}$ and the liquid pressure $p_l$ are locally enriched to better resolve their distribution in the vicinity of cracks by means of an additive decomposition into a large and a small scale part. As far as the representation of cracks is concerned, the Extended Finite Element Method (*XFEM*) is used by enriching the displacement field by means of a jump function as well as crack tip functions. In the framework of the *GFEM* physically motivated enrichment functions for the local enrichment of the $C^1$ discontinuity of the liquid pressure field across cracks are proposed in the paper. The space and time variant analytical solutions obtained from the 1D transient response of saturated porous materials subjected to the liquid pressure within the crack are applied as enrichment functions to locally improve the approximation of the liquid pressure field at discontinuities. Applying these space and time variant functions, which are the exact solutions of the pressure field in the vicinity of cracks, as local enrichment functions lead to a significant improvement in the local approximation of the pressure field at discontinuities. The new *GFEM* model is formulated

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in a poromechanics framework for fully saturated porous materials. Representative analyses demonstrate the improvement of the solution quality compared to existing *FEM* and *XFEM* models.


*Keywords:* Generalized Finite Element Method (*GFEM*); Extended Finite Element Method (*XFEM*); Hydraulic fracturing; Poromechanics

## 1. Introduction

Hydraulic fracturing is frequently used in geoengineering to enhance the mobility of fluids contained in geological layers with low intrinsic permeability. Such processes, denoted as *hydraulic stimulation*, are of vital importance for the exploitation of deep geothermal reservoirs as well as for shale gas and oil reservoirs [1,2]. These processes are characterized by the fluid-induced opening and propagation of fracture zones, which constitute an *a priori* hydro-mechanically coupled problem. Consideration of these propagating fracture zones and their interactions with the fluid flow through the adjacent porous material in numerical analysis using the Finite Element Method (*FEM*) requires strategies to bridge the spatial scale of the fracture and the macroscopic scale (related e.g. to the scale of a typical finite element in poromechanics problems). A standard technique to resolve the small scale of a crack, both with respect to the representation of the displacement discontinuities as well as for the fluid flow, is the use of interface elements (see, e.g. [3]), which, however, requires continuous re-meshing since the cracks follow the topology provided by the element edges [4]. Interface elements have been used in [5,6] for numerical analysis of hydraulic fracturing problems. As an alternative, a local, element-wise enrichment of the displacement field to model $C^0$ continuities of the displacement field within elements has been proposed to represent cracks [7,8], leading to a model formulation resembling Enhanced Assumed Strain methods [9]. This type of models, denoted as Strong Discontinuity Approach (SDA) or Embedded Crack models, requires appropriate crack tracking techniques to realistically predict crack paths [10–12]. The SDA has been extended to hydro-mechanically coupled problems in [13,14] by incorporating a $C^1$ discontinuity of the liquid pressure within an element-wise enhanced pressure formulation. To resolve propagating cracks independently from the underlying (coarse scale) finite element discretization, i.e. without remeshing, the Extended Finite Element method (*XFEM*) [15], exploiting the partition of unity property of shape functions, offers the possibility to include arbitrary enhancement functions into the finite element approximation to improve the local approximation quality of non-smooth distributions of field variables (see, e.g. [15–22] for *XFEM* models for propagating cracks in brittle and cohesive materials or, for shear bands, see [23]). The *XFEM* is a variational multiscale method [24] characterized by a decomposition of the approximation spaces for the relevant field variables as well as for the associated test functions into a coarse scale portion $\bar{\mathcal{S}}$ and a fine scale portion $\hat{\mathcal{S}}$, which is associated with the fine scale resolution in the vicinity of cracks. The function spaces $\hat{\mathcal{S}}$ are chosen according to the physical characteristics of the field variable at the fine scale, i.e. across the cracks. More recently, the *XFEM* has been extended to multi-phase problems [25–28] by incorporating $\mathrm{C}^1$ discontinuous enrichment functions for the liquid pressure field to represent the jump of the fluid flow orthogonal to the crack in addition to $\mathrm{C}^0$ discontinuous enrichment functions for the displacements to represent the fracture zone. $\mathrm{C}^1$ discontinuous distance functions have also been used for the mesh-independent modeling of interfaces along inclusions [29]. Applications to hydraulic fracturing problems are reported in [30–32]. In [30] a *XFEM* model is presented, showing the singular behavior at the crack tip for the different regimes of hydraulic fracturing (toughness dominated, viscosity dominated and the intermediate leak-off-viscosity regime, see [33]) by using a general parametrization of the asymptotic behavior of the enriched displacement field at the crack tip, allowing for a range of asymptotic behavior. The *XFEM* can be regarded as a special variant of the Generalized Finite Element Method (*GFEM*), first introduced in [34] and [35], which combines the standard *FEM* and the Partition of Unity Method (*PUM*) [36,37]. The *PUM* enrichment was first introduced in order to improve the approximation properties over the entire domain in comparison to the standard finite element approximations. Several global enrichment techniques have been proposed based on the *PUM* enrichment, e.g. harmonic polynomials for the Laplace and Helmholtz equations, and holomorphic functions for linear elasticity were investigated. Adopting the enrichment of the approximation space of the standard *FEM* to capture local non-smooth phenomena constitutes a special format of the *PUM* approximation denoted as *XFEM*. This includes $\mathrm{C}^0$ discontinuities of the displacement field in the presence of cracks as well as the $\mathrm{C}^1$ discontinuities in the presence of interfaces. In the *GFEM* proposed by [34] and the *XFEM* [15], the partition of unity is formed by using