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ROBUST ERROR ESTIMATES FOR STABILIZED FINITE ELEMENT APPROXIMATIONS OF THE TWO DIMENSIONAL NAVIER-STOKES' EQUATIONS AT HIGH REYNOLDS NUMBER

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Abstract. We consider error estimates for stabilized finite element approximations of the two-dimensional Navier-Stokes' equations on the unit square with periodic boundary conditions. The estimates for the vorticity are obtained in a weak norm that can be related to the norms of filtered quantities. L^2-norm estimates are obtained for the velocities. Under the assumption of the existence of a certain decomposition of the solution, into large eddies and small fine scale fluctuations, the constants of the estimates are proven to be independent of the Reynolds number. Instead they depend on the L^∞-norm of the initial vorticity and an exponential with factor proportional to the L^∞-norm of the gradient of the large eddies. The main error estimates are on a posteriori form, but for certain stabilized methods the residuals may be upper bounded uniformly, leading to robust a priori error estimates.

Key words. Navier-Stokes' equations, stability, error estimates, high Reynolds number, finite element methods, stabilization, large eddies

AMS subject classifications. 65M12, 65M60, 76F65

1. Introduction.

It is well known that provided the exact solution is sufficiently smooth the approximate solution u_h of the Navier-Stokes' equations on velocity-pressure form can be proved to satisfy estimates of the type

$$\|(u-u_h)(\cdot,T)\|_{L^2(\Omega)} \lesssim e^{\|\nabla u\|_{L^\infty(Q)}} h^{\frac{3}{2}} |u|_{L^2(0,T;H^2(\Omega))}, \tag{1.1}$$

if a consistent stabilized finite element method with piecewise affine approximation is used. Here u denotes the exact solution, h a meshsize, $Q := \Omega \times I$ a space time domain with Ω the space domain and $I := (0,T)$ a time interval with the final time T. We use the notation $a \lesssim b$ for $a \le Cb$ with C a constant independent of the physcial parameters of the problem and the mesh size. We will also use $a \sim b$ for $a \lesssim b$ and $b \lesssim a$. We refer to [12, 6] for examples of analyses of Navier-Stokes' equations on velocity-pressure form and to [17, 18] for analyses on vorticity-streamfunction form. We also give a proof of (1.1) for one of the methods proposed herein in appendix.

Note that there is no explicit dependence on the viscosity in the estimate (1.1). However, for this estimate to be useful the included Sobolev norms must be small, which rarely is the case in the high Reynolds number regime and hence the dependence of the viscosity enters in an implicit manner. If we relax the constraint that the estimate may have no explicit dependence on the viscosity the exponential can be replaced by an exponential of the type $e^{\sqrt{Re}\,T}$, which blows up for vanishing viscosity. The purpose of the present paper is to prove robust error estimates for a subclass of solutions that satisfy a special scale separation property. This is motivated by the results of [11], where it is shown theoretically that the amplitude of the small structures of the flow are exponentially damped for two-dimensional flows. It would therefore seem reasonable that the constant in the estimate (1.1) could be made to depend only on the gradient of the large scales of the flow, since the fine scales are controlled by viscous dissipation. There is still a dependence on the viscosity entering through the scale separation, since as the viscosity goes to zero, so does the energy

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