Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 283 (2015) 177–195
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Topology optimization with pressure load through a level set method

Qi Xia[a], Michael Yu Wang[b], Tielin Shi[a,*]

[a] The State Key Laboratory of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, China
[b] Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, NT, Hong Kong

Received 4 June 2014; received in revised form 15 September 2014; accepted 17 September 2014
Available online 28 September 2014

Abstract

The topology optimization problem with pressure load is solved by using a level set method. The free boundary and the pressure boundary of a structure are represented separately as two zero-level sets of two level set functions, and they are independently propagated during the optimization by solving two Hamilton–Jacobi equations. In order to prevent the two boundaries from touching or crossing each other, the design velocities of the two boundaries that amount to the steepest descent directions are modified. The optimization problem of minimum compliance with perimeter regularization is considered. The shape derivatives of the two boundaries are derived by using the material derivative approach and the adjoint method. The finite element analysis is done through an Eulerian method by employing a fixed mesh and an artificial weak material that represents void. Numerical examples in two dimensions are investigated.

Keywords: Shape and topology optimization; Pressure load; Boundary; Level set method

1. Introduction

The boundary Γ of a structure subjected to pressure load comprises three disjoint segments, i.e., $\Gamma = \Gamma_N \cup \Gamma_D \cup \Gamma_H$. Γ_N is the pressure boundary where a pressure load is applied; Γ_D is the displacement boundary where the structure is supported and the displacements are constrained; Γ_H is the free boundary which is free from both a load and a constraint of displacements. In the optimization of such a structure, all the boundaries can be optimized, and they can be simultaneously optimized. For instance, the shape and topology of both the pressure boundary and the free boundary can be optimized.

It is worthy of note that when the pressure boundary is changed during the optimization, the position and direction of a pressure load change as well [1]. In other words, a pressure load is a design dependent load. Therefore, the optimization with pressure load is a challenging optimization problem. In the literature, this optimization problem is in most cases solved by using the SIMP (Solid Isotropic Microstructure with Penalization) method [2,3]. In the SIMP method, continuous variables representing a material's distribution in a design domain are used to relax the 0–1

* Corresponding author. Tel.: +86 27 87556835; fax: +86 27 87556835.
E-mail addresses: qxia@mail.hust.edu.cn (Q. Xia), yuwang@mae.cuhk.edu.hk (M.Y. Wang), tlshi@mail.hust.edu.cn (T. Shi).

http://dx.doi.org/10.1016/j.cma.2014.09.022
0045-7825/

topology optimization problem to an easier continuous variable optimization problem, and a penalization is applied on intermediate values of the continuous variables to drive the design to a 0–1 or the so-called "black and white" pattern, thus to approximate the discrete nature of a topology optimization problem [2–6]. In addition, numerical schemes, for instance filtering, need to be included in the SIMP method to eliminate numerical instabilities, such as checkerboard pattern and mesh-dependency [7]. Recently, several variants of the SIMP method including the nodal density-based or the pointwise density interpolation-based schemes have been developed, with an aim to overcome these numerical instabilities [8–10]. The SIMP method has got fairly general acceptance because of its conceptual simplicity and easiness of numerical implementation [6].

Nevertheless, difficulties exist in the SIMP based solution to the optimization with pressure load, since one needs to find the pressure boundary from a smooth scalar field that represents the distribution of material in a design domain. As we mentioned above, in the SIMP method, the value 0 of a design variable represents void at a point in the design domain, 1 represents solid, and values between 0 and 1 represent material microstructures [4]. Therefore, the boundary of a structure is only rigorously defined between a 0-region and a 1-region. If the scalar field varies smoothly from 0 to 1 (unfortunately, this is true particularly in the early stage of optimization), the boundary of a structure and the pressure load is ambiguously defined. Actually, several methods were proposed and integrated in the SIMP method to find the pressure boundary and then to apply a pressure load [1,11–14].

On the other hand, several creative approaches that do not need to find the pressure boundary were proposed and integrated in the SIMP based solution. These approaches do not directly apply a pressure load but mimic the effects of a pressure load by artificially incorporating another physical field into the optimization problem, for instance the thermal field proposed by Chen and Kikuchi [15], the fluid field proposed by Sigmund and Clausen [16], Bruggi and Cinquinic [17], the electricity field proposed by Zheng et al. [18]. These approaches open a new way to deal with the pressure load and circumvent the issue of pressure boundary. Nevertheless, the downside of these approaches is that some new numerical issues may arise [16,17] and the optimization is more complex.

The level set method was also used to solve the optimization with pressure load, for example the study by Allaire et al. [19] and by Guo et al. [20]. The level set method [21] is a simple and versatile method for numerical simulation of motion of interfaces, and it has led to a new class of method for structural optimization since the seminal papers [19,22–25]. It directly parameterizes and varies the boundary of a structure, and more importantly it is transparent to topological changes, which is very important for topology optimization and is the key feature that distinguishes it from other boundary variation methods [26,27] for shape optimization. The level set-based method has received much attention in the area of structural optimization, and many efforts have been made to improve it [10,28–31]. In the previous studies of the level set based optimization with pressure load [19,20], the pressure boundary together with the free boundary are represented by the zero level set of one level set function, then the pressure boundary is picked out from the zero level set to apply a pressure load. In [19], the pressure boundary is picked out by checking whether the normal vector is along a specified direction. In [20], the pressure boundary is picked out by performing a sweep operation along a specified direction. Both of the two boundaries are propagated [19,20] by solving one Hamilton–Jacobi equation. In these studies, the pressure boundary is rigorously defined. Nevertheless, when the pressure load comes from several different directions, it will be much more complicated for these approaches to deal with the pressure boundary.

Limitations of the above-mentioned level set based optimization with pressure load lie in the conventional level set method itself, since it is not able to deal with two distinct types of boundaries. Actually, in most of the level set based structural optimizations [19,22–25], only the free boundary is represented implicitly through one level set function, and it constitutes the only design variable of optimization, while the other two boundaries Γ_N and Γ_D are kept fixed during the optimization.

An extension of the conventional level set method, i.e., the multiphase level set method, was proposed to represent multiple types of regions and was used for the topology optimization of heterogeneous structures [32–35]. It is also called the "color level set method" [32]. The idea is to use n level set functions to represent up to 2^n different materials. In the optimization, the free boundary and the interface between different materials of a heterogeneous structure are optimized. However, in these studies, there is no difference in the boundary conditions on the free boundary or the interface between materials, hence being not adequate for the present study.

The objective of our present study is to simultaneously optimize the free boundary and the pressure boundary of a structure subjected to pressure load. The optimization problem is solved by using a novel level set based method proposed in our previous paper [36] that is able to represent multiple types of boundaries. We may call it a "level set based color boundary method". We use two level set functions to represent a structure and two types of boundaries. The