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Finite-strain formulation and FE implementation of a constitutive model for powder compaction

S. Stupkiewicz[a,b,*], A. Piccolroaz[b], D. Bigoni[b]

[a] Institute of Fundamental Technological Research (IPPT), Pawińskiego 5b, 02-106 Warsaw, Poland
[b] University of Trento, via Mesiano 77, I-38123 Trento, Italy

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Highlights

- Constitutive model describing the transition from granular ceramic powder to fully dense state.
- Consistent formulation and computational treatment of elastoplastic coupling at finite strain.
- Yielding described by the 2nd Piola–Kirchhoff stress referred to the intermediate configuration.
- Implicit yield function formulation allowing application of a return mapping algorithm.
- Robust finite element implementation using automatic differentiation technique.

Abstract

A finite-strain formulation is developed, implemented and tested for a constitutive model capable of describing the transition from granular to fully dense state during cold forming of ceramic powder. This constitutive model (as well as many others employed for geomaterials) embodies a number of features, such as pressure-sensitive yielding, complex hardening rules and elastoplastic coupling, posing considerable problems in a finite-strain formulation and numerical implementation. A number of strategies are proposed to overcome the related problems, in particular, a neo-Hookean type of modification to the elastic potential and the adoption of the second Piola–Kirchhoff stress referred to the intermediate configuration to describe yielding. An incremental scheme compatible with the formulation for elastoplastic coupling at finite strain is also developed, and the corresponding constitutive update problem is solved by applying a return mapping algorithm.

Keywords: Plasticity; Elastoplastic coupling; Finite element method; Automatic differentiation

* Corresponding author at: Institute of Fundamental Technological Research (IPPT), Pawińskiego 5b, 02-106 Warsaw, Poland. Tel.: +48 228261281.

E-mail addresses: sstupkie@ippt.pan.pl (S. Stupkiewicz), roaz@ing.unitn.it (A. Piccolroaz), bigoni@ing.unitn.it (D. Bigoni).

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1. Introduction

The formulation and implementation of elastoplastic constitutive equations for metals at large strain have been thoroughly analysed in the last thirty years, see for instance [1,2], so that nowadays they follow accepted strategies. For these materials, pressure-insensitive yielding, J_3-independence, and incompressibility of plastic flow strongly simplify the mechanical behaviour, while frictional–cohesive and rock-like materials (such as granular media, soils, concretes, rocks, ceramics and powders) are characterized by pressure-sensitive, J_3-dependent yielding, dilatant/contractant flow, nonlinear elastic behaviour even at small strain and elastoplastic coupling. There have been several attempts to generalize treatment of metal plasticity at large strain in this context [3–9], but many problems still remain not completely solved. These include the form of the elastic potential, the stress measure to be employed in the yield function, which has to provide an easy interpretation of experiments, the flow rule and the elastic–plastic coupling laws.

The main difficulty in the practical application of finite-strain elastoplasticity models, as opposed to their small-strain counterparts, is related to development and implementation of incremental (i.e., finite-step) constitutive relationships. The difficulties lie, for instance, in formulation and solution of the highly nonlinear constitutive update problem, consistent treatment of plastic incompressibility (or plastic volume changes), and consistent linearization of the incremental relationships. The last issue is of the utmost importance for overall computational efficiency of the finite element models because consistent linearization (consistent tangent) is needed to achieve the quadratic convergence of the Newton method.

In the present paper, the model for cold forming of ceramic powders proposed by Piccolroaz et al. [10,11] (called 'PBG model' in the following) is developed for large strain analyses, implemented in the finite element method and numerically tested. The need for this large-strain generalization is related to the fact that during ceramic forming the mean strain can easily reach 50%, while peaks can touch 80%. The differences between a small strain and a large strain analysis can be appreciated from Figs. 3 and 6 in Section 5.1, where small-strain and large-strain predictions are reported for the force–displacement relation at the top of a rigid mould containing an alumina ceramic powder. Results (pertaining to a flat punch and to a punch with a 'cross-shaped' groove, respectively reported in Figs. 3 and 6) clearly show that the large-strain analyses are more consistent and in closer agreement with experimental results than the analyses performed under the small strain hypothesis.

The model for powder compaction can be considered as paradigmatic of the difficulties that can be encountered in the implementation of models for geomaterials, since many 'unconventional' features of plasticity are simultaneously present to describe the complex transition from a loose granular material (the powder) to a fully dense ceramic (the green body). These difficulties enclose: (i) the pressure-sensitive, J_3-dependent yield function introduced by Bigoni and Piccolroaz [12] ('BP yield function' in the following), which is defined $+\infty$ in some regions outside the elastic domain; (ii) a nonlinear elastic behaviour even at small strain, (iii) changes in elastic response coupled to plastic deformation (elastoplastic coupling).

In this work, incremental (finite-step) constitutive equations are developed and implemented for the finite-deformation version of the PBG model. In order to improve the computational efficiency, the original model [11] is slightly modified, but its essential features, including the elastoplastic coupling, are preserved. Note that a consistent finite-element implementation of the present specific form of elastoplastic coupling at finite strain has not been reported in the literature so far. The model is applied to simulate ceramic powder compaction with account for frictional contact interaction.

The above-mentioned implementation difficulties are efficiently handled by using an advanced hybrid symbolic–numeric approach implemented in *AceGen*, a symbolic code generation system [13,14]. *AceGen* combines symbolic and algebraic capabilities of *Mathematica*, automatic differentiation (AD) technique, simultaneous optimization of expressions and automatic generation of computer codes, and it is an efficient tool for rapid prototyping of numerical procedures as well as for generation of highly optimized compiled codes (such as finite element subroutines). Finite element computations have been carried out using *AceFEM*, a highly flexible finite element code that is closely integrated with *AceGen*.

Selected results of 2D and 3D simulations of powder compaction processes have already been reported in [15], and the model predictions have been compared to experimental data showing satisfactory agreement. However, the finite-strain formulation and the numerical strategies adopted for its implementation have not been presented in [15], as that paper was aimed at providing an overview of elastoplastic coupling in powder compaction processes. In the present paper, we provide the details of the formulation and implementation, and we illustrate the ability of the model