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Adaptive importance sampling for optimization under uncertainty problems

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Highlights

- Optimization under uncertainty problems are discussed using stochastic simulation.
- Adaptive formulation of importance sampling (IS) is considered to improve computational efficiency.
- It is established by sharing information across the iterations of the optimization.
- A sample-based approach is adopted through kernel density approximation.
- Global sensitivity analysis is utilized to select for which model parameters to formulate IS.

Abstract

Design-under-uncertainty problems where a probabilistic performance is adopted as objective function and its estimation is obtained through stochastic simulation are discussed. The focus is on reducing the computational burden associated with the stochastic simulation through adaptive implementation of importance sampling (IS) across the iterations of the optimization algorithm. The proposed formulation relies only on available information (i.e., function evaluations) from the current iteration of the optimization process to improve estimation accuracy in subsequent iterations, and therefore corresponds to a IS selection with a small additional computational burden. Kernel density estimation (KDE) is employed to construct the IS densities based on samples distributed proportional to the integrand of the probabilistic performance. The characteristics of the proposal density are optimally selected to minimize the anticipated coefficient of variation for the objective function if such a proposal density is used as IS distribution. To avoid numerical problems that can occur when trying to develop IS for all uncertain model parameters, a prioritization is first performed using a recently proposed global sensitivity analysis to quantify the relative importance of each model parameter. Therefore, the IS density is only constructed for the most important parameters, with the exact number also a variable that is optimally selected based on the anticipated accuracy. To facilitate the overall adaptive scheme efficient guidelines for the sharing of information across iterations of the optimization algorithm are developed. The numerical example considered verifies the efficiency of the proposed adaptive IS framework.

Keywords: Stochastic optimization; Importance sampling; Kernel density approximation; Optimization under uncertainty; Robust optimization; Simultaneous perturbation stochastic approximation

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Nomenclature

$\mathbf{x}$	Design variables
X	Dismissible design space
n_x	Dimension of $\mathbf{x}$
$\cdot_k$	k^{th} iteration of the optimization algorithm (subscript)
$p(\boldsymbol{\theta})$	Probability density function for $\boldsymbol{\theta}$
$h(\mathbf{x}, \boldsymbol{\theta})$	Performance function
$E_f[.]$	Expectation under probability distribution f (subscript of E)
$Var_f[.]$	Variance under probability distribution f (subscript of *Var*)
$H(\mathbf{x})$	Objective function
N	Number of samples for stochastic simulation
$\hat{H}(\mathbf{x}\|\{\boldsymbol{\theta}^j\})$	Estimate of objective function through stochastic simulation using set $\{\boldsymbol{\theta}^j\}$
$\boldsymbol{\theta}$	uncertain model parameters
Θ	Uncertain Space
n_θ	Dimension of $\boldsymbol{\theta}$
$\cdot_i$	i^{th} component of vector (subscript)
$.^j$	j^{th} sample (superscript)
IS	Importance sampling
$f(\boldsymbol{\theta})$	Importance sampling probability density function
$r(.)$	Importance sampling quotient
$\{\boldsymbol{\theta}^j\}$	Set of samples from $f(\boldsymbol{\theta})$
$\hat{.}$	Estimate using stochastic simulation
δ	Coefficient of variation
$\tilde{\delta}$	Coefficient of variation for new proposal density
δ_{thresh}	Target value of δ
N_{req}	Estimated value of N to establish δ_{thresh}
$\mathbf{w}$	Current subset of $\boldsymbol{\theta}$ for which IS is considered
$q(\mathbf{w})$	Importance sampling density for subset $\mathbf{w}$
n_w	Dimension of $\mathbf{w}$
$\mathbf{y}$	Remaining components of $\boldsymbol{\theta}$ excluding $\mathbf{w}$.
$\tilde{\mathbf{w}}, \tilde{\mathbf{y}}$	Choices for $\mathbf{w}$, $\mathbf{y}$ for next iteration of algorithm
$\tilde{f}, \tilde{r}$	Values for f, r established for $\tilde{\mathbf{w}}, \tilde{\mathbf{y}}$
KDE	Kernel density estimation
$\boldsymbol{\Delta}$	SPSA random perturbation vector
$\mathbf{g}$	SPSA gradient approximation
a	SPSA step size
c	SPSA perturbation magnitude
u	Uniformly distributed random variable
$\pi(\boldsymbol{\theta}\|\mathbf{x})$	PDF proportional to the integrand of the objective function for given $\mathbf{x}$
$\tilde{\pi}(\boldsymbol{\theta}\|\mathbf{x})$	Approximation for $\pi(\boldsymbol{\theta}, \mathbf{x})$ established though KDE
$\hat{\pi}(\boldsymbol{\theta}_i\|\mathbf{x})$	KDE estimation using fix-bandwidth kernel for scalar $\boldsymbol{\theta}_i$
$\boldsymbol{\theta}^a$	Samples from $\pi(\boldsymbol{\theta}, \mathbf{x})$
$\boldsymbol{\theta}^s$	Total number of available samples for Kernel formulation
γ	Relaxation parameter for $\pi(\boldsymbol{\theta}\|\mathbf{x})$ to obtain sufficient samples
σ_i	Standard deviation of i^{th} component of $\{\boldsymbol{\theta}^s\}$
N_{min}	Minimum number of kernel samples obtained in current iteration
$D(.\\|.)$	Relative entropy between the two arguments
D_{min}	Cut-off value for entropy
$\mathbf{d}$	Vector of kernel characteristics
t_i	fixed-window bandwidth parameter for $\boldsymbol{\theta}_i$