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An unfitted Nitsche method for incompressible fluid–structure interaction using overlapping meshes

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Highlights

- Unfitted finite element method for a fluid–structure interaction.
- Proof of stability and accuracy.
- Different coupling scheme's for time advancement: fully coupled or loosely coupled.

Abstract

We consider the extension of the Nitsche method to the case of fluid–structure interaction problems on unfitted meshes. We give a stability analysis for the space semi-discretized problem and show how this estimate may be used to derive optimal error estimates for smooth solutions, irrespectively of the mesh/interface intersection. We also discuss different strategies for the time discretization, using either fully implicit or explicit coupling (*loosely coupled*) schemes. Some numerical examples illustrate the theoretical discussion.

Keywords: Fluid–structure interaction; Incompressible fluid; Unfitted meshes; Fictitious domain method; Nitsche method; Coupling schemes

1. Introduction

The use of Nitsche's method for the coupling of multiphysics problems in computational mechanics has received increasing attention recently (see, e.g., [1]). Thanks to its flexibility and the mathematical soundness it has been used to design methods for several problems. Examples include XFEM of elasticity, i.e., interface problems on unfitted meshes [2,3], and robust and accurate fictitious domain methods [4]. Nitsche's method was first applied to fluid–structure interaction problems in the framework of space–time Galerkin methods in [5] and used to design stable loosely coupled fluid–structure interaction methods in [6,7]. The objective of this note is to apply these techniques

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to design a computational method for fluid–structure interaction on unfitted meshes. The use of fictitious domain or immersed boundary methods for the numerical simulation of fluid–structure interaction problems was pioneered in the works [8–11] and has recently seen a surge of interest in papers such as [12–14].

Our contribution in the present paper is to show how Nitsche's method can be used for fluid–structure computations on unfitted meshes. We consider a low Reynolds regime (Stokes flow) and assume that the structure undergoes infinitesimal displacements (fixed interface). Though simplified, this setting retains some of the main numerical issues that appear in the time-stepping of complex non-linear incompressible fluid–structure interaction problems (see, e.g., [15]). We study both the case where the fluid and the solid equations are defined in domains of $\mathbb{R}^d$ and the case where the fluid is coupled to a thin-walled solid, with the solid equations written in the $(d-1)$-manifold defined by the fluid–solid interface. We present a rigorous proof that optimal convergence in energy norm is obtained for smooth solutions in the space semi-discretized case. Then we discuss different strategies for the time discretization, either fully implicit or the loosely coupled schemes introduced in [6,7].

We will study the case where the solid body is meshed and fitted to the interface. The main motivation for this is the fact that solids are generally described in Lagrangian coordinates and, hence, it is straightforward to move the mesh according to the deformation. This deformed solid mesh is then glued onto the fluid domain without respecting the mesh on the fluid side. This approach is similar to that proposed in [16] for the Poisson problem and more recently in [17] for Stokes' equations. The heterogeneous character of the present system however leads to some difficulties compared to these works. The Nitsche method uses explicit integration of normal stress on the interface to ensure consistency. Since the stress is continuous the stress may be taken from either the solid or the fluid system. Indeed in [16], robustness is ensured by taking all the stresses on the side where the mesh is conforming. This choice is convenient in the homogeneous case, but appears not to be so appealing in the case of fluid–structure interaction. The reason for this is that there is no dissipative mechanism in the solid elastodynamic system that can absorb the perturbation induced by the boundary stresses on the solid side.

If, on the other hand, the Nitsche mortaring is taken only from the fluid side as proposed in [5], the trace inequality necessary for the analysis is no longer robust and the penalty parameter may have to be chosen very large for unfortunate cuts of the mesh, or the resulting system matrix may turn out to be close to singular. In order to nevertheless design a robust and accurate method we suggest to use a ghost penalty term in the fluid (see [18,19] where this technique was proposed for fictitious domain methods for Stokes' problem) in order to extend the coercivity to all of the mesh domain. This is a weakly consistent stabilization term that extends the H^1-stability of the viscous dissipation, as well as the control of the fluid pressure, to the whole fluid mesh-domain, i.e., also to parts of cut elements that are outside the physical domain. We prove that this is sufficient for the method to be stable and optimally convergent.

It should be noted that, even though it appears non-physical, a similar analysis as the one presented below may be carried out if the stresses in the Nitsche coupling terms are taken on the solid side, provided that the solid equations are set in $\mathbb{R}^d$. This requires the use of Gronwall's inequality and leads to the appearance of an unspecified time scale in the Nitsche penalty parameter. Furthermore this approach is not feasible when thin-walled solids are considered and is very inconvenient for nonlinear elasticity. It will therefore not be discussed further herein.

An outline of the paper is as follows. In Section 2 we present a model problem for the fluid–structure interaction and the associated variational formulation. A Nitsche fictitious domain spatial discretization of the problem is proposed in Section 3. Stability and optimal convergence for smooth solutions is proven for this space semi-discretized system in Sections 3.1–3.3. Section 4 is devoted to time discretization, using either implicit or explicit coupling procedures. In Section 5 we present some numerical examples on a two-dimensional model problem. The performance of the fitted and the unfitted methods is compared. We investigate also the accuracy of the explicit coupling schemes proposed. Finally, Section 6 is dedicated to some concluding remarks.

2. A linear model problem

The physical domain consists of $\Omega \stackrel{\text{def}}{=} \Omega^{\text{f}} \cup \Omega^{\text{s}} \cup \Sigma \subset \mathbb{R}^d$, with $d = 2$ or 3, Ω^{f}, Ω^{s} the fluid and solid subdomains, respectively, and $\Sigma \stackrel{\text{def}}{=} \overline{\Omega^{\text{f}}} \cap \overline{\Omega^{\text{s}}}$ the fluid–solid interface. The exterior unit-vectors normal to $\partial\Omega^{\text{f}}$ and $\partial\Omega^{\text{s}}$ are denoted by $\boldsymbol{n}$ and $\boldsymbol{n}^{\text{s}}$. We consider partitions $\partial\Omega^{\text{f}} = \Gamma^{\text{f}} \cup \Sigma$ and $\partial\Omega^{\text{s}} = \Gamma^{\text{s}} \cup \Sigma$ of the fluid and solid boundaries. The fluid is described by the Stokes equations in the polyhedral domain Ω^{f}. For the structure we consider two cases, either the elastodynamics equations in $\Omega^{\text{s}} \subset \mathbb{R}^d$ or the elastodynamic equations for a thin-walled structure (string, membrane or shell) posed on the $(d-1)$-manifold Σ. In the latter case $\overline{\Omega^{\text{s}}} = \overline{\Sigma}$ and $\Gamma^{\text{s}} = \partial\Sigma$.