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Two-scale computational homogenization of electro-elasticity at finite strains

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Abstract

This contribution addresses a two-scale computational homogenization framework for the simulation of electro-active solids at finite strains. A generalized form of the Hill–Mandel condition is employed for the derivation of energetically consistent transition conditions between the scales. The continuum mechanical formulation is implemented into a two-scale finite element environment, in which we attach a microscopic representative volume element at each integration point of the macroscopic domain. In order to allow for an efficient solution of the macroscopic boundary value problem an algorithmically consistent tangent of the macroscopic problem is derived. The method will be applied to the analysis of dielectric polymer–ceramic composites, where we determine the effective actuation of composites with different microstructures. Furthermore, we show the applicability of the proposed method to the computation of two-scale electro-mechanically coupled boundary value problems in consideration of large deformations.

Keywords: Computational homogenization; FE^2-method; Electro-mechanical coupling; Finite deformations; Electro-active polymers; Dielectric composites

1. Introduction

The development of numerical schemes for the simulation of electro-elastic materials undergoing large strains could be helpful for the design and optimization of advanced technical applications in the area of large-strain electro-mechanical actuation like, for example, artificial muscles and robotics [1–5]. The continuum mechanical field theory for the mathematical description of the geometrically nonlinear behavior of these materials is well established [6–10] and has been employed for the modeling of electro-elasticity at finite strains [11–17]. In recent years, numerical solution methods based on different discretization techniques, mainly in the framework of finite element methods,

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have been developed [18–25]. From a practical point of view, materials with very large or "giant" electrostriction are particularly attractive. Thus, the maximization of actuation strains plays an important role in the development of electro-active elastomers. As found out experimentally, the effective electro-mechanical coupling of dielectric elastomers can be enhanced by the addition of high-electric-permittivity (high-ϵ) particles into the elastomeric matrix [26–32]. This phenomenon can be explained by the interplay between the elastomeric matrix and the inclusions, where one notes two major contributions: firstly, the overall electric permittivity is increased due to the addition of high-ϵ dispersion [26]. Secondly, the high contrast of the individual phases on the microlevel induces pronounced electric field fluctuations [33]. This interesting phenomenon is investigated theoretically by using different methods like, for example, homogenization techniques based on sequential laminates [34,35] or computational strategies based on finite element simulations [36,37], see also the works [38–40].

It should be mentioned that the addition of microstructural high-ϵ objects could lead to the important issue of microstructural instability. This can roughly be explained by the large locally induced electric fields in the elastomeric matrix which give rise to pronounced local deformations in the microstructure. In general there are two important phenomena associated with electro-mechanical instability. On the one hand instabilities occur when electrostatic forces cannot be compensated by elastic forces. On the other hand, as in any electrically charged dielectric, there is the danger of electric breakdown. For the case of homogeneous materials these issues are addressed, for example, in [41,42] for static and dynamic conditions; associated experimental observations are documented in [43]. The detailed study of the corresponding effects is however beyond the scope of the present contribution. Thus, the interested reader may be referred to, for example, the works [44–46], in which instability phenomena for composite dielectrics are analyzed in detail.

The present contribution aims at providing a *two-scale computational homogenization framework* for electro-elastic boundary value problems at finite deformations, which can be applied to the simulation, characterization and optimization of electro-elastic solids undergoing large strains. In detail, the framework is based on the FE^2-method, which solves a macroscopic boundary value problem (BVP) in consideration of the response of attached microscopic representative volume elements ($\mathcal{RVE}$). This computational method is well-established in the context of purely mechanical problems [47–58] and has been extended to the homogenization of physically coupled materials like, for example, in thermo-elasticity [59,60] and electro-mechanics [61,62]. Recently, the application to the finite-strain magneto-mechanical case was presented in [63].

The outline of the paper is as follows. In Section 2 the basic kinematical relations and balance equations of finite electro-elasticity will be summarized. Section 3 will provide the two-scale homogenization framework with a focus on the coupled BVPs on the macro- and the microscale as well as on the transition conditions between the two scales. An expression for the macroscopic tangent moduli will be given. In Section 4 the method will be applied to the simulation of polymer–ceramic composites. Section 5 will provide a short summary and a conclusion.

2. Theoretical framework

In the following we summarize the basic equations for the continuum-mechanical description of electro-elasticity at large strains. For a more detailed discussion we refer to the works [13,14] or [18].

Let the body of interest in the (undeformed) reference configuration be denoted by $\mathcal{B} \subset \mathbb{R}^3$ and parameterized in the referential coordinates $\boldsymbol{X}$; in the deformed configuration it is denoted by $\mathcal{S} \subset \mathbb{R}^3$ and is parameterized in the current coordinates $\boldsymbol{x}$. The nonlinear deformation map $\boldsymbol{\varphi}_t : \mathcal{B} \to \mathcal{S}$ at time $t \in \mathbb{R}_+$ maps points $\boldsymbol{X} \in \mathcal{B}$ onto points $\boldsymbol{x} \in \mathcal{S}$. The deformation gradient $\boldsymbol{F}$ is defined by

$$\boldsymbol{F}(\boldsymbol{X}) = \mathrm{Grad}\,\boldsymbol{\varphi}_t(\boldsymbol{X}) = \frac{\partial \boldsymbol{\varphi}_t(\boldsymbol{X})}{\partial \boldsymbol{X}}. \tag{1}$$

In the absence of magnetic fields and free currents the electric field in the current configuration is governed by Faraday's law of electrostatics

$$\mathrm{curl}\boldsymbol{e} = \boldsymbol{0}, \tag{2}$$

so that we can express it as the gradient of some scalar electric potential ϕ with respect to the current coordinates as

$$\boldsymbol{e} = -\mathrm{grad}\,\phi = -\frac{\partial \phi}{\partial \boldsymbol{x}}. \tag{3}$$