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On the discrete variant of the traction method in parameter-free shape optimization

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Abstract

One of the major challenges in the parameter-free approach to computational shape optimization is the avoidance of oscillating (i.e. non-smooth) boundaries in the optimal design trials. In order to achieve this, we investigate a method for regularization that corresponds to the discrete variant of the so-called traction method. In this approach, the design updates are generated in terms of a displacement field, which is obtained as the solution to an auxiliary boundary value problem that is defined on the actual design domain. The main idea herein is to apply fictitious nodal forces corresponding to the discrete sensitivity of the objective function. We propose an algorithm in which constraint functions will be taken into account by using an augmented Lagrangian formulation and a step-length control ensures a sufficient decrease condition in terms of the objective function within each iteration. We examine the benefits of the proposed regularization method on the basis of some numerical examples in comparison to an unregularized steepest-descent algorithm.

Keywords: Shape optimization; Regularization; Parameter-free; Traction method

1. Introduction

In node-based (i.e. parameter-free[1]) shape optimization, the spatial coordinates of nodes located at the boundary of a domain $\mathcal{B}$ serve as design variables for an optimization problem aiming to find a local minimizer of an objective function. The objective function is governed by an elliptic boundary value problem being defined on the domain $\mathcal{B}$. The classification of nodal coordinates as design variables allows for a high flexibility in terms of geometric shape changes, provided the discretization $\mathcal{B}^h$ of the domain is sufficiently fine. The finite element discretization that is used for the analysis of the associated boundary value problem is also the basis for the computation of the shape change. Therefore, no additional representation of the geometric shape is required.

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[1] The term “parameter-free” refers to the approaches in shape optimization in which the design variables are not derived from an existing CAD-parametrization of the model geometry but rather from its finite element discretization.

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Node-based shape optimization problems, provided the existence of an admissible geometry that minimizes the objective function, can be solved by using gradient-based nonlinear programming techniques. Critical issues, however, are the large number of design variables that directly depends on the level of discretization and the occurrence of oscillating boundaries in the optimal design trials.

These issues can be avoided in parametric shape optimization approaches, where the solution of the boundary value problem is still obtained using a finite element approach, but geometric shape variations are described in terms of parameters that define the position of several nodes at once. The parametrization is mostly obtained from a separate geometry model, where geometric features are for instance defined in terms of B-splines or Bézier curves. An overview on parametrization techniques for the application in shape optimization is given in [1]. Drawbacks in using parametric shape optimization approaches are that the flexibility in terms of geometric shape variations is decreased and that an exhaustive preprocessing step is required to obtain the parametrized geometry model. However, other methods have been developed to avoid or at least to dampen oscillating boundaries in shape optimal designs, while still using a parameter-free approach. For example, Meske [2,3] presented a gradient-less shape optimization approach for finite element stress minimization problems. The algorithm is based on a heuristic redesign rule that is derived from physically motivated optimality criteria. The application of the redesign rule leads to an outward-movement of the boundary nodes in high-stress regions and an inward-movement of the boundary nodes in low-stress regions in order to obtain a constant stress level at the design boundary. The development of jagged design boundaries is avoided by using geometric filtering algorithms to smooth the obtained nodal design displacements within each iteration.

Rajan and Belegundu [4,5] proposed a method in which design velocity fields are generated by solving a series of fictitious boundary value problems defined on the actual design domain. The design velocity fields correspond to the respective displacement fields defined by the fictitious equilibria. The fictitious boundary value problems, which are solved by a finite element approach, differ in terms of fictitious loads that are applied at different sections of the design boundary. A superposition of the design velocity fields is used to vary the actual design domain. However, since the parameters that guide the superposition of the design velocity fields are employed as design variables, this approach falls not into the category of node-based shape optimization. To ensure non-oscillating boundaries in the optimal design trials, Zhang [6] extended the fictitious load method by adding beam and plate elements to the fictitious design boundary. Other approaches successfully use nodal coordinates as design variables for gradient-based optimization algorithms. For example, Le [7] presented a length-scale control by filtering the boundary node coordinates using a curve-and-surface-smoothing technique. Therefore, the method prevents artificial contributions of finite element approximation errors to the system stiffness. To avoid excessive mesh distortion, the movement of interior nodes is governed by a Laplacian mesh smoothing algorithm that moves nodal points to the barycentre defined by all adjacent nodal points. All nodes in the set of design nodes are only moved along their respective normal direction in order to reduce the number of design variables. The nodal normal directions are derived from averaging the surface normals of all adjacent boundary faces. Furthermore, a move limit strategy that provides upper and lower bounds for all design variables is employed in order to stabilize the algorithm.

Bletzinger et al. [8–12] proposed an out-of-plane regularization by filter methods to avoid oscillating boundaries in node-based shape optimization of shell structures. Filter methods regularize the sensitivity field and therefore a smooth design update is obtained. The filtering of the sensitivity field is accomplished by the introduction of filter functions that act within an integration domain being specified by a filter radius. Furthermore, an in-plane regularization scheme is used after every shape optimization step to regularize the in-plane mesh distortion. Filter methods are also common in the field of topology optimization, where they provide an effective tool to overcome checkerboarding effects [13].

Allaire [14] presented a shape optimization approach, which is based on the Hadamard method [15,16]. This method can be seen as an application of the gradient method in the field of structural optimization. In addition, in order to avoid optimal designs with oscillating boundaries, two different meshes are initialized. First, a fine mesh is used for the finite element analysis and the computation of the descent direction, which is deduced from the shape gradient. Second, a coarse mesh is extracted from the fine mesh to perform the mesh movement. After every iteration, a mesh adaption deduces a new fine mesh from the updated coarse mesh.

Scherer [17] extended the original shape optimization problem by an energy constraint in terms of a fictitious strain energy measure $\mathcal{I}$. The energy constraint constitutes an inequality constraint that is limited by a maximum allowable energy level $\mathcal{I}^{\max}$, where the energy measure $\mathcal{I}$ represents a fictitious total strain energy that quantifies the shape change of a design proposal with respect to the initial design. One of the major advantages in using this framework is the enhancement of the solvability of the optimization problem due to the fact that optimality criteria are more likely