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Boundary effects in a phase-field approach to topology optimization

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Highlights

- A boundary term is included in the phase-field approach which enables control of outer boundaries.
- The box constraints enforcing the upper and lower limits are included via an obstacle potential.
- The double-sided Howard policy iteration scheme is used for solving the max–min problem.
- An adaptive finite element formulation is used to resolve the interfaces between void and full material.
- The derivation of the stationarity conditions for the objective functional is outlined in detail.

Abstract

A phase-field based topology optimization approach is considered for the maximum stiffness or minimum compliance problem. The objective functional to be minimized consists in addition to the compliance a cost for gray solutions and a cost for interfaces between void and full material. Since the interfaces between void and full material are penalized via a volume integral in the original phase-field formulation there is no penalty associated with interfaces along the external boundaries. In the present contribution, an additional term representing the cost of interfaces at external boundaries is added to the functional subject to minimization. It is shown that the new boundary term enters the optimization as a Robin boundary condition. The method is implemented in a finite element setting and numerical simulations of typical structures are considered. The results indicate that the optimal designs are influenced by the cost of interfaces to a large extent.

Keywords: Topology optimization; Phase-field; Boundary energy; Double obstacle problem; Howard's algorithm

1. Introduction

Topology optimization has over the past decades qualified as an important tool in the design process. The method has evolved and the number of applications is vast, cf. [1,2] for an overview of the method and its applications. The objective of the optimization is in the present work to find a design that maximizes the stiffness for a given amount of material. The advantage of using the stiffness as the objective, or rather its complement the compliance, is that it is a

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global measure and thus can be represented by a scalar value. Moreover, the constraint on the volume is also particular simple since it is linear and monotone which in most cases gives rise to a robust numerical algorithm.

The most widely used numerical scheme for topology optimization is the Solid Isotropic Material with Penalization (SIMP) scheme where the density is approximated as constant within each element. The SIMP procedure is based on a sequence of convex approximations and the algorithm is simple to implement and at the same time numerically efficient. A distinct black/white solution is obtained by penalizing gray designs via a scaling of the elastic constitutive relation. This formulation has been shown to be ill-posed due to the length scale missing in the formulation, cf. e.g. [3]. A mesh-dependent solution can, however, be obtained, by using the finite element method where a length scale enters the formulation via the mesh size. However, to resolve the boundary a very fine or an adaptive mesh is needed, cf. e.g. [4].

Regularization of the topology optimization problem implies that a length scale is introduced in the problem formulation which removes the mesh dependency otherwise present in a finite element solution. Several procedures for regularization of the topology optimization problem has been proposed, these can be local such that the smallest size of a segment is limited, or it can restrict the total length of the perimeter. Such methods are known as filtering techniques [5], perimeter control, [6], gradient control [7,8] and recently also phase-field approaches [9,10]. All these methods has in common that they introduce a length scale in the formulation and thus restricting the smallest characteristic size either locally or globally.

The gradient control method, [7], has been shown to yield similar results as the filter based schemes. The drawback of the gradient control based method is that it implies a large number of additional constraints. The most frequent approach for regularizing the problem is to make use of the filter approach as proposed by [5,11] and further developed in e.g. [12]. A summary of filtering techniques can also be found in [13]. The filter approach is similar to the gradient control method, a local method, however compared to the gradient control method it is associated with a less computational cost. The original filter method is based on that a filtered sensitivity of the density, ρ, is used in the constitutive relation instead of the local density sensitivity. A typical filter can be described as

$$\xi^*(\boldsymbol{x}) = \int_{\Omega_c} \phi(\boldsymbol{x}-\boldsymbol{y})\xi(\boldsymbol{y})dV \tag{1}$$

where the filter function $\phi \geq 0$ fulfills the normalization condition $\int_{\Omega_c} \phi(\boldsymbol{x}-\boldsymbol{y})dV = 1$, Ω_c is a domain in $\mathbb{R}^d$, $d \in [2, 3]$ with compact support. The filter is typically applied directly on the density field or on the sensitivity of the density. Several possibilities for the filter function, ϕ exists and the two most frequently employed filters are the linear and the Gaussian, bell-shaped, cf. e.g. [11,14].

The filter strategy is numerically simple to implement but the treatment of the boundaries deserves additional attention. The difficulties at the boundaries stems from the fact that the filter function, ϕ is defined over Ω_c which close to the boundary stretches outside the design domain, Ω. As a consequence, close to the boundaries of the design domain, $\partial\Omega$, the convolution (1) will involve values of the quantity, ξ, located outside the design domain, Ω. Assuming no material outside Ω and using (1) will inherently lead to a diffuse designs along design boundaries. One remedy (cf. e.g. [15]) is to exclude the part of Ω_c outside Ω along with a scaling of the filter function, ϕ. This procedure tends, however, to yield high values of the density at the interface. In contrast to modifying the filter function, the density field can be extended to the entire, Ω_c, by making use of symmetries. However, all the remedies mentioned above are suffering from having an unclear interpretation at the boundaries.

Another method for regularizing the stiffness problem was proposed in [6] where the objective functional is augmented such that the total variation (TV) of the density field is penalized. The total variation is related to the perimeter of the structure and defined as

$$\mathrm{TV} = \int_{\Omega\setminus\Gamma_j} |\nabla\rho| dV + \int_{\Gamma_j} |\langle\rho\rangle| dS \tag{2}$$

where the element boundaries are defined by Γ_j and $\langle\rangle$ represents the jump function. Since the absolute sign in (2) is non-differentiable at the origin, the problem is regularized using the smoothing functions, g and j, i.e.

$$P = \int_{\Omega\setminus\Gamma_j} g(|\nabla\rho|) dV + \int_{\Gamma_j} j(\langle\rho\rangle) dS. \tag{3}$$