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Stress constrained shape and topology optimization with fixed mesh: A B-spline finite cell method combined with level set function

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Abstract

In this paper, we develop an efficient and flexible design method that integrates the B-spline finite cell method (B-spline FCM) and the level set function (LSF) for stress constrained shape and topology optimization. Any structure of complex geometry is embedded within an extended, regular and fixed Eulerian mesh no matter how the structure is optimized. High-order B-spline shape functions are further implemented to ensure precisions of stress analysis and sensitivity analysis. Meanwhile, level set functions, i.e., implicit functions are used to enable topological changes of the considered structure through smooth boundary variations. Involved parameters rather than the conventional discrete form of LSF are directly taken as design variables to facilitate the numerical computing process. To be specific, the LSF is constructed by means of R-functions that incorporate cubic splines as implicit functions to offer flexibilities for shape optimization within the framework of fixed mesh, while the compactly supported radial basis functions (CS-RBFs) are employed as implicit functions for stress constrained topology optimization. It is shown the proposed FCM/LSF method is a convenient approach that makes it possible to calculate stress and stress sensitivities with high precision. Representative examples of shape and topology optimization with and without stress constraints are solved with success demonstrating the advantages of the FCM/LSF method.

Keywords: Shape optimization; Topology optimization; B-spline finite cell method; Level set function; R-functions; Stress constraints

1. Introduction

Shape and topology optimizations were extensively studied and achieved remarkable progress in the past three decades. Shape optimization is traditionally based on the parametric representation of structural boundaries in terms of computer aided design (CAD) curves or surfaces. Boundaries are smoothly modified without topological changes for the local stress reduction while Lagrangian mesh is used to follow the boundary variation. Comparatively, topology optimization is to find the optimal material distribution over the specific design domain. It is generally applied at

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the conceptual design stage with a fixed Eulerian mesh. Global responses such as the mean compliance and natural frequencies are minimized by means of the solid isotropic material with penalization (SIMP) method. Unfortunately, mesh updating and even mesh distortion constitute the main obstacle in traditional shape optimization due to large shape modifications especially for structures of complex geometries. As to topology optimization, stress-based designs are important but relatively difficult due to the lack of boundary smoothness. Recently, some appealing works [1,2] have the tendency to combine both kinds of optimization methods with the merits of each other. An ideal way is to unify shape and topology optimization into such a design procedure that takes into account stress constraints and utilizes Eulerian fixed mesh simultaneously.

Attempts were made to employ the Eulerian fixed mesh in shape optimization for the avoidance of laborious remeshing process and mesh distortion. For example, Kim and Chang [3] proposed to interpret the boundary moving effect as the density variations of cutting boundary elements similar to the boundary element homogenization. Jang and Kim [4] used piecewise oblique lines to approximate the boundary in the calculation of stiffness matrices of cutting boundary elements. However, it was found that the parametric representation of a structure boundary was inconvenient to identify and handle cutting boundary elements in the two above methods. An alternative way is to apply the level set function (LSF) or level set method (LSM) [5–7] into the Eulerian-type structural optimization where the moving boundary is tracked by means of implicit LSFs of discrete form. Owing to its advantage of easily identifying relative positions of finite elements, the LSF was rapidly recognized in the community of structural optimization [8–11]. Chen et al. [1] constructed the LSF by means of R-functions [12–14] consisting of implicit parametric functions of primitives to favor the shape representation of free-form nature. In this way, manufacturability, topological changes and analytical sensitivity analysis are made possible. Miegroet and Duysinx [2,15] combined the extended finite element method (X-FEM) with the implicit LSF of discrete form transformed from the non-uniform rational B-splines (NURBS) curve of explicit parametric form for shape optimization. Although one such explicit–implicit transformation makes ease the definition of shape design variables with NURBS control points, the derivation of analytical sensitivity seems to be difficult.

Nowadays, stress constrained topology optimization is a motivating subject because it interprets the practical need of engineering designs rather than the standard compliance-based topology optimization [16]. As the latter is mainly based on the SIMP method, it is hard to reduce stress intensities accurately due to the lack of boundary smoothness and the presence of gray elements. In certain cases, stress singularity associated with the element removal should also be taken into account [17]. The implementation of conventional LSF of discrete form [9,11] realized topology optimization by changing boundaries of embedded holes. Although this strategy can greatly improve the boundary smoothness, it has limitations in the solution of Hamilton–Jacobin equation related to Courant–Friedrichs–Lewy (CFL) condition, upwind schemes, extension velocities and reinitialization algorithms [18]. To remedy this, globally supported radial basis functions (GS-RBFs) were introduced by Wang and Wang [19]. Luo et al. [20] further introduced compactly supported radial basis functions (CS-RBFs).

In practice, LSF is often implemented in combination with the X-FEM [21,22] to guarantee the applicability of fixed mesh and the stress computing accuracy. Due to the huge number of local stress constraints, a surrogate model was often defined by a single global constraint or a relatively small number of block aggregated constraints [23–26] such as Kreisselmeier–Steinhauser (K–S) function and P-norm. Moreover, Le et al. [27] introduced an adaptive normalization coefficient scheme to approximate the maximum stress. Zhang et al. [28] enhanced global stress measures with boundary curvatures and stress gradient. Wang and Li [29] constructed a concise shape equilibrium constraint function with both merits of low computational cost and precise local control.

In this paper, we develop an integrated FCM/LSF design framework. Based on the concept of finite cell method (FCM) [30–35], any structure of complex geometry to be optimized is embedded within a fictitious regular domain that is discretized by a fixed mesh of regular cells. The adapted integration scheme is further used to deal with cutting boundary elements. Here, the B-spline FCM [36–39] is extended for shape and topology optimization to achieve a high-order continuity and stress accuracy along cell boundaries with less degrees of freedom. Comparatively, the simplified version of X-FEM [18,22,28,29,40] can be regarded as a variant of the B-spline FCM with shape functions of order one. Furthermore, CS-RBFs and R-functions are implemented for topology and shape optimization. Structural boundaries are described by implicit cubic splines to offer a great design flexibility with only coordinates of a few number of interpolation points being design variables. Weighted extended B-spline (Web-spline) [41,42] is adapted to the implementation of homogeneous Dirichlet boundary conditions because the LSF representing the structural boundary can easily be revised as weight functions to B-spline basis functions in the interpolation of displacement field.