Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 278 (2014) 388–403
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Discrete double directors shell element for the functionally graded material shell structures analysis

M. Wali*, A. Hajlaoui, F. Dammak

Mechanical Modelization and Manufacturing Laboratory (LA2MP), National Engineering School of Sfax, B.P W3038, Sfax, University of Sfax, Tunisia

Received 24 December 2013; received in revised form 28 March 2014; accepted 7 May 2014
Available online 11 June 2014

Abstract

In this paper, the accuracy and the efficiency of the 3d-shell model based on a double directors shell element for the functionally graded material (FGM) shell structures analysis is studied. The vanishing of transverse shear strains on top and bottom faces is considered in a discrete form. Thus, the third-order shear deformation plate theory (TSDT) is a particular case of the discrete double directors shell model (DDDSM) used in the present work. The DDDSM is introduced to remove the shear correction factors, when using the first-order shear deformation theory (FSDT), and improve an excellent performance when compared with other works. This model can be used for static, free vibration and buckling analyses of FGM. The convergence of the proposed model is compared to other well-known formulations found in the literature.

Keywords: FGM; Shell element; Third-order deformation theory

1. Introduction

In recent years, shell structures made of FGMs are widely used in many engineering fields such as aerospace, gas turbines, nuclear fusions, electronics, etc. because they present many advantages. Indeed, they are used in systems which need high heat-resistance, high rigidity and eventually absence of the interface problem unlike laminate structures. The material properties of FGMs are inhomogeneous and vary continuously in one or more directions. Typical FGMs are made from a mixture of ceramic and metal, or a combination of different metals or different ceramics that are appropriate to achieve the desired objective.

The importance of this kind of materials motivates many contemporary researchers to study their properties and behaviors. Among these studies, we mention works of Vel and Batra [1] who presented the three dimensional exact solution for free and forced vibrations of functionally graded rectangular plates, Ferreira et al. [2] who studied the static deformations of functionally graded plates using the radial basis function collocation method and a higher-order shear deformation theory, they selected the shape parameter in the radial basis functions by an optimization procedure based on the cross validation technique.

Matsunaga [3] calculated the natural frequencies and buckling stresses of plates made of FGMs using a 2-D higher-order deformation theory. Carrera et al. [4] evaluated the effect of thickness stretching in plate/shell structures made by FGM in the thickness directions. Neves et al. [5] presented a quasi-3d hyperbolic shear deformation theory for the

* Corresponding author. Tel.: +216 20385960; fax: +216 74666535.
E-mail address: mondherwali@yahoo.fr (M. Wali).

http://dx.doi.org/10.1016/j.cma.2014.05.011

bending and free vibration analysis of functionally graded plates. Xiang et al. [6] used an n-order shear deformation theory and a meshless global collocation method based on the thin plate spline radial basis function to analyze the static characteristics of functionally graded plates under sinusoidal load.

The study of FGM structures using classical theory, based on the Kirchhoff hypothesis, is lack of precision. The inaccuracy is due to neglecting the effects of transverse shear and normal strains of the structure. In order to take into account the effects of gradual change of material properties, the first-order shear deformation theory (FSDT) and higher-order shear deformation theories (HSDT) have been used in the analyses of FGMs. However, since using the FSDT, shear correction factors should be incorporated to adjust the transverse shear stiffness and the accuracy of solutions will be strongly dependent on the correction factors. Examples of the FSDT approach are given in [7,8].

To analyze static and dynamic behavior of FGMs, a number of theoretical formulations and finite element models based on the HSDT were developed. Reddy [9] presented a general formulation for FGMs using the third-order shear deformation plate theory and developed the associated finite element model that accounts for the thermo-mechanical coupling and geometric non-linearity. Numerous works using HSDT are published to study the transverse shear deformations through the FGM shell thickness, such as [10–12].

On the other hand, to model the multi-layered structures capable to take into account the strong discontinuities in material properties across the thickness, a multi-directors shell theory has been used in the literature. In this approach, the theories of shells are considered as oriented 2D areas with additional kinematic variables modeling the shell behavior. Such continuous domain is known as Cosserat surface [13]. In the same context, Başar et al. [14] developed a refined finite-rotation theory with seven independent displacement variables for arbitrary multilayered shell structures made particularly of composite material layers. This theory approximates the displacement field by a cubic series expansion of thickness coordinates, which imply a quadratic shear deformation distribution across the thickness. Başar et al. [15] presented a multi-directors shell theory on the basis of a quadratic approximation of the displacement field. Their contribution is the development of four-node isoparametric shell elements providing an accurate prediction of interlaminar stresses in composite laminates; special attention is given to the consideration of finite rotations as well as large deformations. An enriched kinematic field in order to extend the potential application domain of the shell model is developed by Brank and Carrera [16], Brank et al. [17] and Brank [18]. In these works, the researchers discussed a theoretical formulation of shell model accounting for through-the-thickness stretching, which allows large deformations and direct use of 3d constitutive equations.

The primary objective of this work is to study FGM shell structures. The problem formulation is established from an adopted 3d-shell nonlinear model based on a double directors shell element. However, this paper treats only linear FGM shell structure by linearizing equations obtained from the developed non-linear DDDSM formulation with a third-order deformation theory. The linearized model is validated both by standard tests and by comparing the accuracy and the performance with works in the literature. In fact, the used double directors shell model is developed in [19] to study constant elastic materials and extended in this paper for FGM shell structures. The third-order shear deformation plate theory of Reddy [20] and Reddy [9], is the linear version of the DDDSM model projected in a 2d xy-plane. In the modeling, the vanishing of transverse shear strains on top and bottom faces is considered in a discrete form, similar to the development of the nonlinear discrete Kirchhoff shell element presented in the work of Dammak et al. [21].

The outline of the paper is as follows. In Section 2, the material properties of the functionally graded materials are illustrated. In Sections 3 and 4 the kinematic of the double directors shell model and the weak form of shell equilibrium equations are presented respectively. The implemented finite elements are described in Section 5. A number of numerical simulations are presented in Section 6 and closing remarks are stated in Section 7.

2. Material properties

An FGM shell structure with polynomial material law, as given by Zenkour [22], is considered. The shell structure is graded from aluminum (bottom surface) to alumina (top surface) materials. The following functional relationship is considered for the Young modulus $E_{\text{FGM}}(z)$ in the thickness direction:

$$E_{\text{FGM}}(z) = E_m + (E_c - E_m)\left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{1}$$

where $E_m = 70$ GPa and $E_c = 380$ GPa are the Young modulus of the metal and ceramic components, respectively and n is the power-law index. The Poisson ratio for both metal and ceramic is assumed to be constant and equal to $\nu = 0.3$.