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An APDM-based method for the analysis of systems with uncertainties

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Abstract

In this work a novel approach, called the Surface Reference Method (SRM), is proposed for the analysis of discretized systems with uncertain properties. It is based on the result of the Approximated Deformation Principal Modes (APDM) method by Falsone and Impollonia and on a simple property of the uncertain response of systems modeled as specified in this paper, that is: in the space of the random variables the response varies linearly along the straight lines passing by the origin. Using this property, an approximate method is obtained based on some new coefficients that enable to improve the APDM method.

This new method enables analyzing systems with very high level of uncertainty with a low computational effort. In order to evidence the goodness of the method, two numerical examples are proposed considering several types of distributions for the uncertain parameters.

Keywords: Uncertain systems; Probabilistic approaches; Probability density function

1. Introduction

The analysis of structural systems is often carried out through the use of deterministic methods, although it is known that the uncertainties due to the characterization of materials imply that the response system would be better represented as random. Assuming the mechanical properties of the system as random variables, an assessment of the response in terms of probability is required and it needs to use advanced specific calculation methods (probabilistic methods). In the literature there are several papers related to the application of probabilistic methods and, over the years there have been significant results obtained in this field.

It is known that there is no universal method to resolve any problem involving uncertainty. A useful overview of methods for the study of systems with uncertainty is given in [1–8].

Perhaps the oldest method for the evaluation of the probability density function (pdf) of the response is the cumulant series expansion of the characteristic function [2]. This method provides good results if the response is not strongly

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non-Gaussian. When the response is strongly non-Gaussian, the number of terms of the series may be particularly high and the convergence, that is not guaranteed, can be particularly slow. In addition, the direct evaluation of the terms of the series may not be simple. For this reason this method is often associated with the Monte Carlo (MC) simulation method [3,9,10].

Widely used methods are those based on the perturbation approaches, based on a Taylor series expansion in terms of a set of zero mean random variables. The perturbation approaches provide accurate results for low level of uncertainty (corresponding to few terms of the series). Unfortunately, when the level of uncertainty of the structural parameters increases the approach loses strongly its precision and, moreover, the computational effort increases exponentially because a high number of terms of the series must be taken into account. Major details on this method are given in [1,11–15].

Another important class of methods for solving uncertain structural systems is that of projection approaches, based on the projection of the solution on a complete stochastic basis. The simplest projection approach is the Karhunen–Loève expansion [7]. One of the most used approach is that based on the polynomial chaos expansion, that is a Galerkin projection scheme based on Wiener integral representation [1,7]. The approach requires to evaluate numerically the terms of the series expansion [16,17]; it can be particularly onerous if the terms of the series are not limited to a relatively small number, even if several efforts have been made to improve the approach [18,19]. A comparison of different projection schemes for stochastic finite element analysis is given in [20].

Recently a new approach has been proposed, based on a new version of the Probabilistic Transformation Method (PTM), for the study of some stochastic problems [21,22]. The method is still in its infancy but provides the basis for a new philosophy of analysis of systems with uncertainties or subjected to random loads, providing exact solutions in some cases, and approximated in other cases. In the context of uncertain parameters in [23] an approximate method is proposed to estimate the random response in terms of displacements, using the approximate expression of the structural response given in [24]. The method provides an elegant solution in terms of pdf response, but it suffers the limits of the approximation on structural response and on the fact that only the pdf of the displacements can be evaluated. The elegance of the approach pays off in the lower versatility.

Another interesting category of methods dealing with uncertain system are those based on the random matrix expansion of the structural stiffness matrix in order to perform explicitly its inversion [25,26]. Then, once that the explicit inverse stiffness matrix is known, it is possible to obtain the statistics of the response, or to perform an MC simulation to obtain the response pdf.

As said before, there is no universal method for the analysis of systems with any type of uncertainty. The only universal instrument for the analysis of systems with uncertainties is the direct MC simulation [27,28]. It has the advantage of enabling the analysis of any system having any type of randomness, but has the disadvantage of the high computational effort, so that it is limited to structures with a small number of DOFs. This has led several authors to propose MC-based methods, more versatile than other methods but having a computational cost lower than that required by the direct MC method. This category includes the above mentioned MC-based statistical approach and the above mentioned expansion methods of the structural stiffness matrix when they are used together with the MC simulation [9,10,24–30]. To reduce the computational effort of the MC method, in [30] the structural response is obtained explicitly with respect to the uncertain parameters, enabling to overcome the inversions of the stiffness matrix; however the formulation is heavy and the implementation of the method is too complicated for very large systems.

In 2002 Falsone and Impollonia [29] proposed the method of analysis APDM belonging to the class of MC-based methods. It consists in breaking up the structural response on the basis of the main deformation modes of the structure: this allows obtaining an approximation of the response, without the cost to invert the stiffness matrix of the system, enabling to reduce strongly the computational effort performing the response by MC simulation. In a certain sense, the method enables even to evaluate an approximation of the inverse stiffness matrix (like the matrix expansion method); from another point of view it consists in the expansion of the structural response on a particular base (like the projection method) of a finite number of functions depending on the uncertain parameters (principal deformation modes), where, however, the coefficient of the series can be evaluated explicitly in terms of the uncertain parameters. Moreover, the approach is not based on perturbation, as remarked in [1]. This method, despite its remarkable efficiency, allows the analysis of systems with relatively low levels of uncertainty (but sufficiently high if compared with the usual range required in engineering fields).

The aim of this work is to propose a method for the stochastic analysis of finite element modeled uncertain structures able to provide accurate results even for a level of uncertainty higher than that enabled by the APDM. This