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A new higher-order finite volume method based on Moving Least Squares for the resolution of the incompressible Navier–Stokes equations on unstructured grids

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Abstract

In this work a new higher-order (>2) accurate finite volume method for the resolution of the incompressible Navier–Stokes equations on unstructured grids is presented. The formulation is based on the use of Moving Least Squares (MLS) approximants. Third and fourth order accurate discretizations of the convective and viscous fluxes are obtained on collocated meshes. In addition, MLS is employed to design a new Momentum Interpolation Method that allows interpolations better than linear on any kind of mesh. The accuracy and performance of the proposed method is demonstrated by solving different steady and unsteady benchmark problems on unstructured grids.

Keywords: High-order methods; Incompressible Navier–Stokes equations; Moving Least Squares; Momentum Interpolation Method; Unstructured grids; Collocated grids

1. Introduction

Even though all fluids are compressible in an absolute sense, incompressible flow approximation can be assumed when the flow speed is small enough compared with the speed of sound of the medium. Numerical solutions of the incompressible Navier–Stokes equations have a great interest due to its wide range of applications in the areas such as low speed aerodynamics, biomedical fluid flow and hydrodynamics.

The main problem with numerical solutions of incompressible flow is the difficulty in coupling changes of the velocity field with changes in the pressure field while satisfying the continuity equation. The reason is the absence

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of a transient term in the continuity equation, so the equations are decoupled and the continuity equation can be considered as a constraint that the velocity field has to satisfy.

Pressure based methods, the artificial compressibility method and methods based on derived variables are the three main approaches for solving the incompressible Navier–Stokes equations. The Marker And Cell (MAC) method [1] was probably the first method for the solution of incompressible flow using a derived Poisson equation for the pressure in order to satisfy mass conservation. In this approach the pressure is used as a mapping parameter to satisfy the continuity equation. The method of artificial compressibility proposed by Chorin [2], introduces a pseudo-time derivative of the density into the continuity equation. Physically, this means that waves of finite speed are introduced into the incompressible flow field as a medium to distribute the pressure. Numerically the pseudo-term changes the character of the continuity equation from elliptic to hyperbolic. This allows the system of equations to be solved with a variety of time marching schemes developed for compressible flow solvers. However, the artificial compressibility method introduces a delay between the flow disturbance and its effect on the pressure field, whereas in a true incompressible flow, the pressure field is affected instantaneously by a disturbance. The major drawback of this method is the need for the definition of an artificial compressibility parameter, that is specific for each problem. The original form of Chorin's method was developed for steady state problems, and Peyret and Taylor [3] extended it to a time accurate formulation. Later, the method was fully extended to general three dimensions by Kwak et al. [4].

In order to remove the pressure from the formulation, different approaches introducing other variables instead have been developed. The most common approach of this kind is the stream function vorticity method [5]. The extension of this approach to 3D problems adds much complexity to the formulation and is more expensive than methods which solve velocity and pressure.

There are usually two kinds of grid arrangements used to solve the incompressible Navier–Stokes equations: staggered grids and non-staggered (collocated) grids. On the staggered grids, variables are stored at different locations shifting the half of a control volume in each coordinate direction. The main advantage of this approach is that no interpolation is required, since the variables are stored where they are needed. However, the staggered arrangement is difficult to apply for unstructured and curvilinear grids. This difficulty increases when one deals with 3D problems. On the other hand, in collocated grids, vector variables and scalar variables are stored at the same locations, usually in the centroid of the control volume. The main problem of this approach is the possibility of checker-board due to the central-difference discretization of the pressure. This weakness was circumvented by Rhie and Chow [6] that proposed a momentum-based interpolation method to interpolate mass fluxes on cell faces. In their approach, they mimic a staggered-grid discretization by expressing the discrete mass conservation equation in terms of the discrete mass fluxes across cell faces. The Rhie and Chow approach has been widely used for decades [7–9]. The basic idea of the Momentum Interpolation Method (MIM) is to remove the interpolated pressure gradient term that comes from the momentum equation and then add the pressure gradient calculated directly at cell face. Majumdar [10] and Miller [11] reported that solutions of steady-state problems from Rhie and Chow original MIM are dependent on the underrelaxation factor. In order to eliminate this dependency Majumdar proposed an iterative algorithm that its implementation can achieve a unique solution. Choi in 1999 [12] reported that the solution using the original MIM scheme is time step size dependent and proposed a modified Momentum Interpolation Method. In 2002 Yu et al. [13] observed that the solutions obtained with Choi's scheme are still time step size dependent, though the dependence is quite small. Yu et al. [13] proposed a different interpolation technique and they demonstrated numerically and mathematically that the resulting scheme was time step size and underrelaxation factor independent. The original formulation has been very recently improved to solve these problems in [14].

The Semi Implicit Method for Pressure Linked Equations, better known as the SIMPLE algorithm, is a pressure correction method. It uses the pressure as a mapping parameter in order to satisfy the decoupled equations. The main advantage of this approach is that the Poisson equation is not solved, since it is approximated with a pressure correction equation. Comparison between the staggered grid and collocated grid [12,15] shows that the SIMPLE-like algorithms on collocated grids can provide similarly accurate results and convergence rates as those on staggered grid [16]. The SIMPLE method and its versions are currently widely used to solve the incompressible Navier–Stokes equations. Further details of the method and its variants can be found in [17].

Although second-order accurate methods are currently the standard in industry for computations of incompressible flow on unstructured grids, high-order discretization techniques offer the potential to significantly reduce the computational costs for a given accuracy by reducing the mesh size. Alternatively they can be employed to increase the accuracy for a fixed grid size compared to lower-order methods.