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A hybrid approach for the time domain analysis of linear stochastic structures

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ABSTRACT

A new hybrid approach for the time domain analysis of linear stochastic structures with uncorrelated or correlated random variables is proposed. This new hybrid approach combines the modal approach, the second-order perturbation technique and the number theoretical method (NTM). In the hybrid approach, an approximate stochastic model of the finite element (FE) model is developed in the sense of first-order accuracy, which provides second-order estimations of the mean and covariance matrices of structural responses. Compared with the FE model, the proposed model is more convenient in terms of the computation effort it requires. By employing the NTM to evaluate the statistical moments of solutions of the approximate model, the secular terms, contained in the results of the perturbation stochastic finite element method, are eliminated. Two numerical examples are presented to demonstrate the accuracy and efficiency of the method proposed.

1. Introduction

In engineering, the dynamic analysis of stochastic structures is quite important. The dynamic responses of a true structure are influenced by various random factors such as unpredictable external excitations, random material parameters, random geometric properties, and so on. For a deterministic structure, many accurate and efficient approaches have been developed, e.g., the finite element method (FEM). However, these deterministic approaches cannot be employed in the stochastic case, which requires stochastic approaches. Usually, the use of random field approaches in combination with the well-known FEM is referred to as the stochastic finite element method (SFEM). A number of approximate methods [1] have been proposed to solve stochastic systems. These approximate methods can be roughly categorized as being based on the Monte Carlo simulation method (MCS), the spectral stochastic finite element method (SSFEM) or the perturbation stochastic finite element method (PSFEM).

Among the proposed stochastic approaches, MCS is the most widely used method. This method is applicable to any stochastic problems. When MCS is used for a stochastic structure, a number of deterministic analyses need to be performed first, which requires considerable computational effort. Recently, some developments have been proposed in the literature. In [2,3], the parallel MCS was developed. In [4,5] the method that combines the subspace iterative method with MCS was proposed to address the eigenproblems of stochastic structures. In [6], the Neumann expansion was introduced in conjunction with MCS to perform dynamic analysis of stochastic structures in the frequency domain. In [7], the Neumann MCS was employed to perform time domain analysis of stochastic systems. Although many enhancements to MCS have been proposed, it is still too time-consuming. In SSFEM, another type of stochastic method, the random field is usually discretized using the Karhunen–Loève expansion, and the structure's nodal displacements are approximated using a polynomial chaos (PC) expansion [1,8]. The computational cost of SSFEM is smaller than that of MCS. However, the computational effort increases exponentially with the order and number of uncertain quantities involved, which puts some practical restrictions on this method. In [9], a numerical solution of SSFEM was presented. In [10], a condensation technique was suggested to reduce the computational cost of SSFEM and compute the non-stationary random vibration response of an uncertain linear system. In [11], hybrid perturbation-PC approaches to random eigenproblems were proposed. The literature to date on SSFEM has addressed mainly static stochastic problems.

PSFEM, which is also widely used due to its computational efficiency, is aimed primarily at stochastic structures with low-level uncertainties. Fortunately, for many real engineering structures with random material and geometric parameters, the coefficients

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of variation of these random parameters are usually small. As with SSFEM, studies on PSFEM have mainly addressed static stochastic problems. The results of time domain analyses of stochastic systems using PSFEM contain secular terms. Hence, PSFEM is often used to perform frequency domain analyses of stochastic systems. In [12], PSFEM was employed to perform the modal analysis of propellers. An approach based on PSFEM and mode superposition was suggested in [13] to analyze linear stochastic systems in the time domain. In [14], PSFEM was applied to the time domain analysis of nonlinear stochastic structures. The state variable vector was introduced in [15] in conjunction with PSFEM for the time domain analysis of stochastic structures. None of these studies has addressed the problem of secular terms. The generalized nth PSFEM was developed in [16,17] to improve the accuracy of PSFEM. However, the computational effort of this method increases exponentially with the order and number of uncertain quantities involved. In [18], a hybrid approach that combines the perturbation technique, modal approaches and MCS was developed for use in dynamic analysis of stochastic structures in the frequency domain. A drawback of this method is that the probability density function (PDF) of structure frequencies need to be guessed in terms of the means and variances of frequencies.

There are other approaches available for addressing stochastic problems. An approach to static analysis [19,20] and frequency domain analysis [21] of linear stochastic systems has been proposed that can yield the explicit relationship between the system response and random parameters. A method based on generalized probability density evolution and the number theoretical method (NTM) was proposed in [23–25]. In [26–28], the pseudo excitation method was developed for the frequency domain analysis of structures with random excitations. This method is not applicable to a stochastic system with random material and geometric parameters. Reviews of these stochastic approaches can be found in [1,22].

A limited amount of research on the time domain analysis of stochastic structures has been conducted. MCS and SSFEM are often too time-consuming. PSFEM is limited because of the presence of secular terms in the results. In this paper, a new approach that combines the perturbation technique, the number theoretical method (NTM) [29–31] and the modal approach is proposed for analysis of stochastic dynamic systems with low-level uncertainties. This hybrid approach can produce second-order estimations of the mean and covariance matrices of structural responses and can remove the secular term. In the next section, the proposed method is described in detail. A summary of the proposed method and its algorithm are presented in Section 3. In Section 4, two numerical examples are presented. The numerical results are compared with results obtained using MCS and PSFEM.

2. The proposed method

2.1. The basic idea

Suppose that the model of a linear stochastic dynamic system produced by the finite element method (FEM) can be written as

$$\boldsymbol{M}(\boldsymbol{\varepsilon})\ddot{\boldsymbol{u}}(t)+\boldsymbol{C}(\boldsymbol{\varepsilon})\dot{\boldsymbol{u}}(t)+\boldsymbol{K}(\boldsymbol{\varepsilon})\boldsymbol{u}(t)=\boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}, \boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively. $\boldsymbol{F}$ is the load vector. $\boldsymbol{u}, \dot{\boldsymbol{u}}$ and $\ddot{\boldsymbol{u}}$ are the displacement, velocity and acceleration vectors, respectively. t is the time. $\boldsymbol{\varepsilon}=\{\varepsilon_i\}$ is the random vector containing q zero-mean random variables, where q is the number of random variables. The standard deviation of ε_i is denoted by σ_i. If the perturbation method is employed to solve an undamped dynamic system, the solution can be expanded as

$$\boldsymbol{u}(\boldsymbol{\varepsilon},t)=\boldsymbol{u}_0(t)+\sum_{i=1}^{q}\boldsymbol{u}_{1,i}(t)\varepsilon_i+\sum_{i=1}^{q}\sum_{j=i}^{q}\boldsymbol{u}_{2,ij}(t)\varepsilon_i\varepsilon_j+O(\varepsilon_i^3) \tag{2}$$

According to the perturbation theory [35], $\boldsymbol{u}_{1,i}(t)$ contains several terms multiplied by t that lead to $|\boldsymbol{u}_{1,i}(t)|\to+\infty$ for $t\to+\infty$. These terms are usually called the secular terms. However, damping is useful in eliminating the secular terms. In this study, to overcome the negative influences of the secular terms, a hybrid approach is proposed. The basic idea of the proposed method here is the use of the second-order perturbation technique and the modal approach to produce an approximate model of the FEM model. Compared with the original model governed by Eq. (1), the approximate one is more convenient to compute and can provide the second-order estimates of the mean and covariance matrices of structural responses.

2.2. The modal approach

Suppose further that $\boldsymbol{C}$ is a type of proportional damping matrix. The modal approach can be applied to Eq. (1). Let us define the eigenproblem of such a stochastic dynamic system as

$$\boldsymbol{K}(\boldsymbol{\varepsilon})\boldsymbol{x}_k(\boldsymbol{\varepsilon})=\lambda_k(\boldsymbol{\varepsilon})\boldsymbol{M}(\boldsymbol{\varepsilon})\boldsymbol{x}_k(\boldsymbol{\varepsilon}) \tag{3}$$

$$\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{M}\boldsymbol{x}_l=0,\quad k\neq l \tag{4}$$

in which $\boldsymbol{x}_k$ and λ_k are the kth eigenvector and eigenvalue, respectively. We know that the displacement can be expressed as

$$\boldsymbol{u}=\sum_{k=1}^{N}a_k\boldsymbol{x}_k \tag{5}$$

where N is the number of degrees of freedom (DOF). Substituting Eq. (5) into Eq. (1), we have

$$\sum_{k=1}^{N}\ddot{a}_i\boldsymbol{M}(\boldsymbol{\varepsilon})\boldsymbol{x}_i+\sum_{k=1}^{N}\dot{a}_i\boldsymbol{C}(\boldsymbol{\varepsilon})\boldsymbol{x}_i+\sum_{k=1}^{N}a_i\boldsymbol{K}(\boldsymbol{\varepsilon})\boldsymbol{x}_i=\boldsymbol{F} \tag{6}$$

Multiplying both sides of Eq. (6) by $\boldsymbol{x}_k^{\mathrm{T}}$ in conjunction with Eq. (4) produces

$$m_k\ddot{a}_k+c_k\dot{a}_k+k_ka_k=f_k \tag{7}$$

where

$$\begin{aligned}&m_k=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{M}(\boldsymbol{\varepsilon})\boldsymbol{x}_k,\quad c_k=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{C}(\boldsymbol{\varepsilon})\boldsymbol{x}_k\\&k_k=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{K}(\boldsymbol{\varepsilon})\boldsymbol{x}_k,\quad f_k=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{F}\end{aligned} \tag{8}$$

The initial conditions are

$$a_k(t_0)=\boldsymbol{x}_k^{\mathrm{T}}\boldsymbol{u}(t_0),\quad \dot{a}_k(t_0)=\boldsymbol{x}_k^{\mathrm{T}}\dot{\boldsymbol{u}}(t_0) \tag{9}$$

in which t_0 represents the initial time. In terms of Eq. (5), the mean and covariance matrices of the displacement can be written as

$$E(\boldsymbol{u})=\sum_{k=1}^{N}E(a_k\boldsymbol{x}_k) \tag{10}$$

$$\mathrm{cov}(\boldsymbol{u},\boldsymbol{u})=\sum_{k=1}^{N}\sum_{l=1}^{N}\mathrm{cov}(a_k\boldsymbol{x}_k,a_l\boldsymbol{x}_l) \tag{11}$$

It can be found from Eqs. (10) and (11) that $\boldsymbol{x}_k$ is necessary for evaluating the mean and covariance of the displacement vector. In the next section the second-order perturbation technique is introduced for determining the eigenvector $\boldsymbol{x}_k$.

2.3. The second-order estimates of eigenvectors

First, let us expand the mass, damping and stiffness matrices via Taylor series as