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A high order time-accurate loosely-coupled solution algorithm for unsteady conjugate heat transfer problems

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ABSTRACT

Thermal interaction of fluids and solids, or conjugate heat transfer (CHT), is encountered in many engineering applications. Noting that time-accurate computations of transient CHT problems can be computationally expensive, we consider the use of high order implicit time integration schemes which have the potential to be computationally more efficient relative to the commonly used second order implicit schemes. For thermally weak couplings, we present a loosely-coupled solution algorithm where high order implicit–explicit (IMEX) Runge–Kutta schemes are employed for time integration. The IMEX schemes consist of the explicit first-stage singly diagonally implicit Runge–Kutta (ESDIRK) schemes, for advancing the solution in time within each separate fluid and solid subdomain, and the explicit Runge–Kutta (ERK) schemes, for explicit integration of part of the coupling terms. By considering a numerical example (an unsteady conjugate natural convection in an enclosure), temporal order preservation of the coupling algorithm (without subiterating) is demonstrated. In addition, the stability of the loosely-coupled algorithm is investigated numerically for the CHT test-case; when the ratio of the thermal effusivities of the fluid and solid subdomains is much smaller than unity, using large Fourier numbers of the subdomains is possible, indicating that time-step size is restricted by accuracy rather than stability. Furthermore, the (computational) work-(temporal) precision character of several time integration schemes in solving the CHT test-case is compared over a range of accuracy requirements; for time-accurate solutions, the fourth and fifth order IMEX schemes are 1.5 times more efficient than Crank–Nicolson and 2.7 times more efficient than BDF2. The computational gain is higher for smaller tolerances.

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1. Introduction

Thermal interaction of flows and structures, also referred to as conjugate heat transfer, arises in many engineering applications. Examples are: the thermal interaction among various phases and materials in the mold region of a continuous casting process, the thermal interaction between the fluid in a U-tube pipe with the surrounding soil in a geothermal heat exchanger system, the cooling of gas turbine blades, and the cooling of electronic chips. In order to obtain a better understanding of the physics of the coupled problem and hence to increase the efficiency and/or safety of designs, numerical simulations serve as a viable tool.

The monolithic and partitioned approaches are two commonly used methods for solving the thermal coupling of flows and structures. In the monolithic method, the solution in the global domain is obtained by solving the governing equations within the subdomains as well as the interface equations simultaneously [1,2]. The monolithic method requires the production of a single code specifically tailored for conjugate heat transfer problems [3]. In the partitioned method on the other hand, a separate physics solver is associated with each subdomain [4]. The solution in the subdomains are coupled at the interface through a set of transmission conditions, and a coupling algorithm is required for the transfer of data between the subdomains. By solving the coupled problem in a partitioned manner, one can take advantage of the already existing efficient and highly optimized separate fluid and solid codes.

In engineering applications, typically an implicit time integration is preferred over an explicit one in order to circumvent time step restrictions due to probable stiffness in the problem. Stiffness in a system can, for example, arise due to the nature of the governing equations or due to the generated grid (such as clustering of nodes near an area of interest [5]). Performing time-accurate computations of transient CHT problems can be computationally demanding, in particular when low order time integration schemes are used. The obtained solution can suffer from low levels of temporal accuracy, and in order to increase the accuracy of the solution, smaller time steps must be taken. This results in an increase in the computational cost of solving the coupled problem. As a potential solution, we consider the use of high order implicit time integration schemes for advancing the coupled problem in time.

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Computational efficiency of the high order time integration schemes relative to second order backward difference scheme (BDF2) has been demonstrated in Bijl et al. [6] for fluid flow computations and in van Zuijlen and Bijl [7] for the partitioned simulation of the mechanical coupling of flows and structures. Using high order spatial discretization in combination with high order temporal discretization can also contribute to the overall efficiency of the method. In this paper however, following the method of lines approach, we will assume a spatial discretization, and focus on time integration.

When the coupled problem is solved monolithically, using an implicit time integration scheme, the interface equations are resolved implicitly. However, in the partitioned method, some or all of the interface terms are treated explicitly, depending on the arrangement with which the two coupled domains are solved (parallel (Block Jacobi) or sequential (Block Gauss–Seidel)). Therefore, in the partitioned approach, the interface equations are treated in a segregated manner where one of the interface equations is applied as a boundary condition for one subdomain and the other as boundary condition for the second subdomain [4]. If at each time step (or stage of an implicit Runge–Kutta scheme), a single interface iteration (subiteration or fixed-point iteration) [4,8] is performed, the partitioned algorithm is referred to as loosely-coupled, otherwise it is referred to as strongly-coupled. This paper focuses on loosely-coupled solution algorithms; see [4,9] for examples on strongly coupled algorithms, and [3,4,10–12] for examples on partitioned algorithms with explicit time integration.

Loosely-coupled solution algorithms can provide an efficient way of solving time-accurate CHT problems relative to the monolithic approach when the thermal coupling between the subdomains is weak (when the ratio of the thermal effusivities ($e = \sqrt{k\rho c_p}$) of the coupled domains is much smaller than unity) [13]. To the authors' knowledge, loosely-coupled solution algorithms with up to second order implicit time integration schemes have been reported in the literature (see [3,9,13,14]). In this paper, a loosely-coupled solution algorithm is presented in which a family of high order implicit–explicit (IMEX) Runge–Kutta schemes are used for time integration. The IMEX schemes consist of the explicit first-stage singly diagonally implicit Runge–Kutta (ESDIRK) schemes, which are used for advancing the solution in time within each separate fluid and solid subdomain, and equal order and number of stages explicit Runge–Kutta (ERK) schemes for explicit integration of part of the coupling terms. The IMEX schemes considered here were originally developed for solving time-accurate convection–diffusion-reaction (CDR) problems [15] and later employed for the loosely coupled simulation of the mechanical coupling of flows and structures [7].

The solution obtained using a loosely-coupled algorithm, contains an additional source of error compared to the monolithic solution, denoted as the partitioning error. As a result, the temporal accuracy and stability of the coupling algorithm is influenced by the partitioning error. Therefore, in designing loosely-coupled solution algorithms, a number of issues needs to be considered. One, whether the design order of the time integration scheme is preserved *without subiterating*. Second, what are the stability properties of the algorithm; for practical computations, it is preferred that Δt is restricted by accuracy rather than stability. These two issues are investigated numerically by considering a CHT problem (unsteady conjugate natural convection in an enclosure).

While in the loosely coupled multi-stage IMEX schemes a single interface iteration is performed at each (implicit) stage, in the second order loosely coupled Crank–Nicolson scheme [13], only one is performed per time-step. However, for the same time-step, the high order IMEX schemes generally provide temporally more accurate solutions. For the CHT test-case, the (computational) work-(temporal) precision character of the high order IMEX and second order Crank–Nicolson and BDF2 schemes is compared over a range of accuracy requirements. We investigate whether the high order IMEX schemes can compete with the second order schemes for a reasonable portion of the work-precision spectrum, i.e. whether the additional work per time-step of the IMEX schemes is compensated by the gain in temporal accuracy.

In what follows, first the equations governing conjugate heat transfer are discussed. After a brief overview of the ESDIRK and IMEX time integration schemes, the details of the loosely coupled solution algorithm are presented. Next, numerical examples are considered, in order to demonstrate the applicability of the algorithm, to investigate the temporal order preservation and stability of the algorithm, and finally its computational efficiency relative to the second order time integration schemes.

2. Governing equations

In the conjugate heat transfer problem considered here, the fluid domain is modeled using the Boussinesq approximation of the Navier–Stokes system which in primitive variables is given by:

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = -(\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \nu\nabla^2\boldsymbol{u} - \nabla p - \beta g \boldsymbol{j}(T_f - T_{ref}), \tag{2}$$

$$\frac{\partial T_f}{\partial t} = -(\boldsymbol{u}\cdot\nabla)T_f + \frac{1}{(\rho c_p)_f}\nabla\cdot(k_f\nabla T_f), \tag{3}$$

where $\boldsymbol{u}$ is the velocity vector, p is the kinematic pressure, ν the kinematic viscosity, T_f the temperature, k_f the thermal conductivity, $c_{p,f}$ the heat capacity, ρ_f the density, β the compressibility factor of the fluid, g the acceleration due to gravity, and $\boldsymbol{j}$ is a vector indicating the direction in which the gravity acts.

The solid domain is modeled using unsteady heat conduction:

$$\frac{\partial T_s}{\partial t} = \frac{1}{(\rho c_p)_s}\nabla\cdot(k_s\nabla T_s). \tag{4}$$

The governing equations are accompanied by appropriate initial and boundary conditions. For a well-posed problem, the continuity of the temperature and heat flux are imposed at the common interface ($\mathcal{I}$) of the domains:

$$T_f(\mathcal{I},t) = T_s(\mathcal{I},t), \tag{5}$$

$$q_s(\mathcal{I},t) = q_f(\mathcal{I},t), \tag{6}$$

where $q_m(\mathcal{I},t) = -k_m\nabla T_m\cdot\boldsymbol{n}$ with m the index of the subdomain and $\boldsymbol{n}$ the outward normal of the interface.

To identify the governing parameters in the conjugate heat transfer problem described by (1)–(6), the equations are non-dimensionalized using appropriate non-dimensional quantities (see [13]). Based on the dimensionless form of the equations, it is observed that the Prandtl number ($Pr = \frac{\nu}{\alpha_f}$), the Rayleigh number $\left(Ra = \frac{g\beta(T_H - T_C)L_{ref}^3}{\nu\alpha_f}\right)$, the ratio of the thermal conductivities ($\frac{k_s}{k_f}$), and the ratio of thermal diffusivities of the domains $\left(\frac{\alpha_s}{\alpha_f}\right)$ are the governing parameters of the problem.

3. Model problem

In this section, the description of a one dimensional model problem is presented which will be used to discuss the details of the loosely-coupled solution algorithm. The model problem has been commonly used in the literature (for example [3,4,10]) to analyze stability of numerical algorithms for thermal coupling of domains. The model problem consists of thermal coupling of two domains $\Omega_1 = [-L_1, 0]$ and $\Omega_2 = [0, L_2]$, with their common

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