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Scattering from targets in three-dimensional littoral and surf-zone environments with multi-layered elastic sediments based on an interior-transmission formulation

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ABSTRACT

We present a new numerical approach to compute the three-dimensional scattering of acoustic and elastic waves from elastic objects buried in multi-layered elastic sediments. We use a frequency-domain approach and utilize an elastodynamics formulation based on an interior-transmission representation of the scattering problem. This enables us to use the perfectly-matched-layer approximation (PML) method for exterior truncation while avoiding some fundamental problems associated with the truncation of the non-homogeneous exterior domains with coupled elasto-acoustics. We present several examples that verify the new formulation and its finite-element-based numerical implementation. We also demonstrate its application to important problems involving the computation of scattered fields from buried elastic objects buried in littoral or surf-zone domains with a multi-layered sediment modeled as three-dimensional visco-elastic material.

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1. Introduction

The ability to accurately compute the three-dimensional scattering of acoustic and elastic waves by elastic obstacles fully or partially buried in sediment is important in a number of areas. In military applications, it has a critical role in mine-countermeasures; in the civilian realm, it is important in the ongoing development of approaches to detect unexploded ordnance based on the structural-acoustic features contained in the scattered signals. The domain for problems of interest here is described in Fig. 1. It consists of an environment with a sediment (bottom) halfspace that is layered. For deep littoral environments, the top halfspace may be described in terms of a single semi-infinite water layer; for shallow littoral environments, the top halfspace may consist of a finite water layer topped by a semi-infinite air layer; for surf-zone environments, where the unsaturated sediment can range from dry to wet, the top halfscape consists entirely of a semi-infinite air layer. One or more visco-elastic objects, with general three-dimensional geometry, are fully or partially embedded in the sediment. Our goal is to compute the scattering of incident acoustic plane-waves traveling from the top halfspace in shallow littoral or surf-zone environments.

The mathematical representation of the sediment plays a critical role in the computed scattering response. For littoral applications where the sediment is fully saturated, it can be modeled effectively as a damped acoustic fluid. Several publications have addressed this problem by treating the sediment as a (multi)-layered fluid domain. An axisymmetric model for simulating sonar interrogation of targets near sediments is presented in [1]. It couples a normal-mode representation in the farfield combined with a finite element based formulation in the nearfield. Another axisymmetric finite-element-based approach is presented in [2]. Recently, a more comprehensive, fully three-dimensional model with multiple fluid layers is discussed in [3].

While the published models admit geometrically and materially complex targets, they are inadequate for modeling situations that involve a surf-zone environment where the sediment is either dry or wet (partially saturated). This is the case because a fluid-based sediment model can only represent compressional waves and stresses. It excludes several important physically-observed response-features for dry or unsaturated sediments, particularly those that are due to shear-stresses or rely on three-dimensional elasticity such as the *Possion-effect*.

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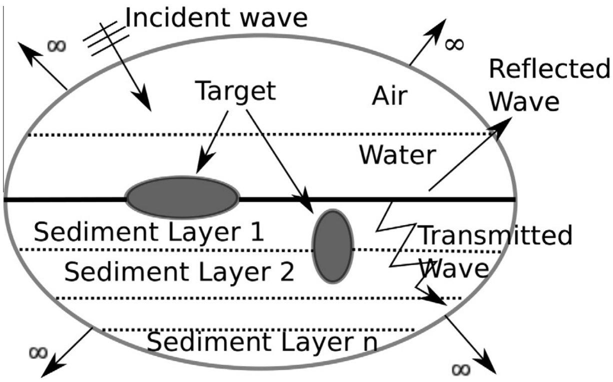

Fig. 1. Physical description of problems of interest.

Additionally, our modeling is intended to support detection of buried objects using techniques based on shearography.[1] It requires the model to compute the full three-dimensional elastic displacement and strain field on the fluid–sediment interface. This can only be done if the sediment is modeled as a fully three-dimensional (visco)-elastic material. Others have developed methods for scattering from cylindrical or other canonical target shapes in multi-layered elastic media based on combining finite element and integral-equation methods [4]. In addition to being limited to canonical shapes for the scatterers, they are not easily scalable for large problems due to the global coupling of all the unknowns on the truncation boundary.

Based on the above considerations, we need to address the following two technical issues. Firstly, we need an exterior scattering formulation that treats the sediment as a multi-layered (visco)-elastic material and allows for ensonification by incoming plane-waves. Secondly, since the domain of interest includes the infinite half-spaces for the fluid and the sediment domains (see Fig. 1), one must address the effective truncation of the infinite exterior for computational purposes.

For computation truncation of the infinite exterior, three main approaches exist: *Infinite-elements* (IE) [5,6], *artificial/absorbing boundary conditions* (ABC) [7,8], or *perfectly-matched-layer approximation* (PML) [9]. When the infinite exterior is homogenous, IE methods have been shown to be highly effective [5,6]. However, because they are derived from asymptotic expansions for outgoing waves in a homogeneous freefield, they are not applicable to inhomogeneous exterior. ABC methods suffer similar limitations. The PML approach, reformulated based on complex coordinate stretching [10], offers a very general approach for exterior domain truncation that is naturally compatible with finite element based discretizations. It has been successfully used for littoral cases where the saturated sediment is modeled as a damped acoustic fluid [3,11] as well as geophysics problems for modeling propagation in inhomogeneous infinite elastic domains [12].

In this paper we present an approach that treats the air above the sediment as a general viscoelastic material. We point out that doing so does not make our approach inaccurate, invalid, or unrealistic. While one might argue that it may be a computational overkill to model air as a general (visco) elastic material, it is a more general model than a scalar fluid model in terms of including all the dynamical physics. More importantly, modeling the air in the upper-halfspace as a general (visco) elastic material enables us to use the PML approach. This is *crucial* because even though PML-based approaches have been applied to non-homogenous exterior domains, they are usually effective only when the exterior domain is modeled using single physics (same governing equations everywhere). Our numerical experiments, discussed later in this paper, demonstrate that use of PML can lead to an inconsistent approximation with large errors when the interface between the fluid and elastic subdomains, governed by different physics (equations), penetrates into the PML region(s).

While PML-based approaches for exterior elastodynamics have been used before in geophysical applications [12], for the most part they mostly model cases involving *propagation* not scattering where all source(s) of excitation are confined in the interior of the domain of interest and well away from the truncation boundaries. This is different than scattering problems that involve excitation by incident waves coming from infinity in the upper (fluid) halfspace. Furthermore, a straightforward exterior elastodynamic formulation that ensonifies the entire fluid–sediment interface will not work because the source of excitation will then extend all the way into the PML truncation region leading to large errors. We employ an interior-transmission formulation for elastodynamic scattering that ensures that all sources of excitation for the scattering problem are in the interior of the domain well away from the PML region.

The rest of this paper is organized as follows: Section 2 describes the model problem; it points out issues with PML when used with a coupled fluid–elastic formulation, and then describes an alternative formulation based on an interior transmission formulation for elastodynamics. Section 3 first presents several numerical examples to verify the formulation followed by applications to problems with scattering from elastic objects buried in the multi-layered elastic sediments. We conclude with a summary of the current work and future extensions in Section 4.

2. Model problem

Consider the computational domain described in Fig. 2 where Ω^e_j and Ω^f_k represent layer (s) of the sediment and fluid subdomains, respectively. Ω^s_i represents the scatterer volume (s). $\Gamma_{ij} = \Omega^e_i \cap \Omega^s_j$ represents the boundary of the scatterer j in contact with sediment (fluid) layer i. Of course we allow the scatterer(s) to intersect multiple layers including the fluid–sediment boundary Σ.

We seek the time-harmonic (frequency-domain) linear response of such systems to acoustic waves incident from the fluid subdomain. The equations governing the scalar acoustic pressure ϕ and the vector sediment displacement $\mathbf{u}$ are given by

$$L^f(\phi) = 0 \text{ in } \Omega^f_k \quad (1)$$
$$L^e(\mathbf{u}) = 0 \text{ in } \Omega^e_i \cup \Omega^s_j \quad (2)$$

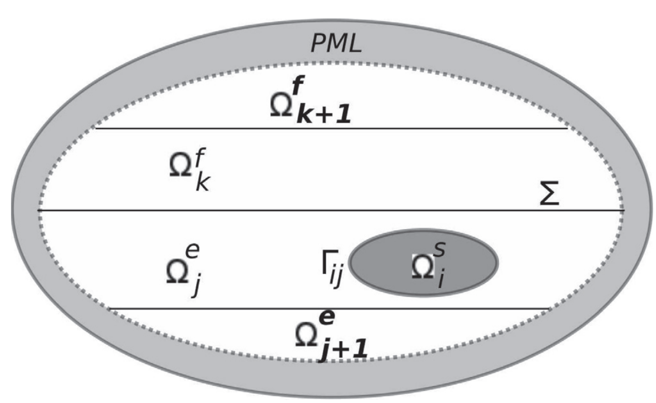

Fig. 2. Computational domain for problems of interest truncated by a PML-layer. The fluid and the elastic half-spaces each may be layerwise heterogeneous.

[1] Methods requiring a three-dimensional strain field or, more generally, displacement gradients.