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Electromechanical instability on dielectric polymer surface: Modeling and experiment

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ABSTRACT

We present a dynamic finite element formulation for dielectric elastomers that significantly alleviates the problem of volumetric locking that occurs due to the incompressible nature of the elastomers. We accomplish this by modifying the Q1P0 formulation of Simo et al. [1], and adapting it to the electromechanical coupling that occurs in dielectric elastomers. We demonstrate that volumetric locking has a significant impact on the critical electric fields that are necessary to induce electromechanical instabilities such as creasing and cratering in dielectric elastomers, and that the locking effects are most severe in problems related to recent experiments that involve significant constraints upon the deformation of the elastomers. We then compare the results using the new Q1P0 formulation to that obtained using standard 8-node linear and 27-node quadratic hexahedral elements to demonstrate the capability of the proposed approach. Finally, direct comparison to the recent experimental work on the creasing instability on dielectric polymer surface by Wang et al. [2] is presented. The present formulation demonstrates good agreement to experiment for not only the critical electric field for the onset of the creasing instability, but also the experimentally observed average spacing between the creases.

1. Introduction

Dielectric elastomers (DEs) are a class of soft, active materials that have attracted significant attention in recent years [3–8]. They have been found to provide excellent overall performance in actuation-based applications, including high specific elastic energy density, good efficiency and high speed of response. Furthermore, DEs are typically lightweight, flexible and inexpensive materials which makes them ideal candidates for high performance, low cost applications where fabrication of the DEs into a wide range of shapes and structures can easily be realized [9].

While DEs have been found to exhibit good performance with respect to a variety of actuation-relevant properties, including strain, actuation pressure, efficiency, response speed, and density [10], the key source of the technological excitement surrounding DEs stems from the fact that, if sandwiched between two compliant electrodes that apply voltage to the elastomer, the DE can exhibit both significant thinning and in-plane expansion. This unique large deformation-based actuation capability has led to many interesting applications for DEs, including the potential to harvest energy from sources as diverse as human muscle motion and ocean waves, medical devices, and perhaps most importantly, artificial muscles [3,4,7]. Furthermore, recent experimental studies by Wang et al. [11] and Shivapooja et al. [12] have exploited the large deformation and surface instabilities studied in DEs in the present work to generate dynamic surface patterns, and antifouling coatings, respectively. In both of these cases, it is the large deformation and instability of the polymer that enables the novel applications, which may not be achieved with traditional electroactive materials. In addition, the voltage required to deform the polymer scales with the thickness of the polymer, which therefore may not be very high for thin polymer films.

Due to these and other potentially groundbreaking applications, starting about 15 years ago with the seminal work of Pelrine et al. [10,13], there have been many experimental studies to elucidate the electromechanical behavior and properties of DEs [14–25].

Along with the experimental studies, many analytic theories that explain various aspects of the electromechanical behavior and properties of DEs have recently been developed [26–33,20,21]. Many of these theories have as their basis the original works in electro-elasticity, for example that of Maugin [34]. Furthermore, there have recently appeared a range of analytical studies on the stability and instability phenomena both in DEs [35–38], and other magneto-elastic materials [39].

While these analytic theories have led to many key insights regarding the electromechanical behavior and instabilities of DEs, it has been difficult to use these analytic theories to study the

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inhomogeneous deformation and failure mechanisms, i.e. wrinkling [19,21], electromechanical snap-through instability [13], and more recently creasing and cratering instabilities [2,40,11] that have been observed experimentally.

Because of this, several papers have recently appeared proposing finite element (FEM) formulations for DEs [41–44,33,45–49]. The approaches of Zhao and Suo [41] and O'Brien et al. [44] are similar in that neither formulation accounted for the full electromechanical coupling, i.e. electrostatic effects were accounted for via inclusion in the mechanical free energy, while no electrostatic governing equation was solved. The approaches of Vu et al. [42] and Zhou et al. [43] are similar in that both utilized finite deformation, fully coupled electromechanical equations that were solved neglecting inertia. While the work of Vu et al. [42] did not consider electromechanical instabilities, such effects were considered by Zhou et al. [43], though difficulties in tracking the entire history of the electromechanical instability were found due to the static formulation. Wissler and Mazza [45] solved the coupled electromechanical problem using Poisson's equation for the electrostatics, though again, electromechanical instabilities were not considered. Recently, Park and Nguyen [46] and also Khan et al. [47] proposed viscoelastic FEM models for DEs.

Overall, there exist two major unresolved issues in the existing FEM modeling literature. First, none of the previous approaches have demonstrated the ability to capture inhomogeneous deformation and failure modes (creasing, cratering, snap-through, wrinkling) that result from the electromechanical instability within a large deformation framework. Second, none of the previous approaches has been able to resolve the electromechanical instabilities while ensuring that the incompressible nature of the material response, and thus avoidance of volumetric locking effects, is accounted for.

The first issue was resolved through a recent FEM formulation proposed by Park et al. [50], who utilized inertia to capture electromechanical instabilities that arise through the constitutive model and field equations of Suo et al. [26]. Inertia is important for this approach as quasistatic FEM techniques, without special techniques such as the arclength method, fail once the loss of ellipticity (corresponding to softening in the voltage-charge curve for DEs) occurs. In contrast, the use of inertia enables the simulation to continue into the electromechanical softening regime, as demonstrated by Park et al. [50] and Park and Nguyen [46]. The role of inertia in electromechanically coupled problems is thus exactly analogous to its role in single field mechanical strain softening problems [51]. Using the dynamic formulation, they were able to demonstrate the basic electromechanical instabilities that occur in DEs under electrostatic loading, i.e. snap-through instabilities, surface wrinkling and creasing.

However, the work of Park et al. [50] did not resolve the second issue, i.e. that of volumetric locking that arises due to the incompressible material response of the DEs. As discussed by Belytschko et al. [52], FEM modeling of volumetric locking has a lengthy history, though the salient point is that the vast majority of the literature has been targeted towards single-field (i.e. mechanical-only) problems. In the present work, we extend one such approach to alleviating volumetric locking, the classic three-field Hu–Washizu Q1P0 formulation of Simo et al. [1], to problems involving coupling of the mechanical and electrostatic domains. We note that while the viscoelastic formulation of Park and Nguyen [46] also utilized the Q1P0 formulation of Simo et al. [1] to alleviate volumetric locking, an explicit comparison to experimental results to demonstrate the necessity and accuracy of the electromechanical Q1P0 formulation was not performed. Because of this, we also demonstrate the capability of the proposed approach in accurately capturing the experimentally observed critical electric fields needed to induce electromechanical instabilities, as well as the experimentally observed spacing by Wang et al. [2]. Comparisons are also made to standard three-dimensional linear and quadratic hexahedral FEs to demonstrate the utility of a specialized formulation to alleviate volumetric locking effects.

2. Background: nonlinear electromechanical field theory

The numerical results we present in this work are obtained using a FEM discretization of the electromechanical field theory recently proposed by Suo et al.[26], and recently reviewed by Suo [27]. In this field theory, at mechanical equilibrium, the nominal stress s_{iJ} satisfies the following (weak) equation:

$$\int_V s_{iJ}\frac{\partial \xi_i}{\partial X_J}dV = \int_V \left(B_i - \rho\frac{\partial^2 x_i}{\partial t^2}\right)\xi_i dV + \int_A T_i\xi_i dA, \tag{1}$$

where ξ_i is an arbitrary vector test function, B_i is the body force per unit reference volume V, ρ is the mass density of the material and T_i is the force per unit area that is applied on the surface A in the reference configuration.

For the electrostatic problem, the nominal electric displacement $\tilde{D}_I$ satisfies the following (weak) equation:

$$-\int_V \tilde{D}_I\frac{\partial \eta}{\partial X_I}dV = \int_V q\eta dV + \int_A \omega\eta dA, \tag{2}$$

where η is an arbitrary scalar test function, q is the volumetric charge density and ω is the surface charge density, both with respect to the reference configuration.

We make several relevant comments with regards to the field equations in (1) and (2). First, if the vector test function ξ_i is chosen to represent a virtual displacement δu_i, the mechanical weak form in (1) represents the well-known statement of virtual mechanical work, where the nominal stress S_{iJ} is work conjugate to the gradient of virtual displacement δu_i. Second, if the electrical test function η in (2) is chosen to be the virtual potential $\delta\phi$, then the electrostatic weak form in (2) can also be interpreted within a virtual work context, where the nominal electric displacement $\tilde{D}_I$ is work conjugate to the gradient of virtual potential $\delta\phi$. Third, the strong form of the mechanical weak form in (1) is the well-known momentum equation, while the strong form of the electrostatic weak form in (2) is the well-known Gauss's law.

Because we are solving an electromechanical boundary value problem, it is relevant to discuss the details of the boundary conditions for each field equation. Specifically, the electromechanical boundary conditions are in fact the standard boundary conditions for each of the single domain problems. Specifically, these are applied tractions and displacements for the mechanical domain and applied voltages and charges for the electrostatic domain. No non-standard boundary conditions are needed in the present formulation.

We also note that the weak formulations in (1) and (2) do not account for the possible effect of the surrounding free space. However, both for the problem we analyze in the current work, as well as the vast majority of experimental DE configurations, the DE is actuated by coating it with electrodes and the effect of electric fields around the edges of the electrodes is negligible. For other situations in which an air gap exists between one electrode and the polymer, it is likely that the electric field in the air will have a significant effect on the instability mechanism [53].

We note that the theory of Suo et al. [26] is not the only nonlinear electromechanical field theory that exists; earlier works by Dorfmann and Ogden [29,30] and McMeeking and Landis [31] also proposed nonlinear electromechanical field theories for deformable elastomeric materials. For this work, we utilize the governing nonlinear electromechanical field equations of Suo et al. [26] for the following five reasons: (1) The field variables (mechanical