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# Distance fields on unstructured grids: Stable interpolation, assumed gradients, collision detection and gap function

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ABSTRACT

This article presents a novel approach to collision detection based on distance fields. A novel interpolation ensures stability of the distances in the vicinity of complex geometries. An assumed gradient formulation is introduced leading to a $C^1$-continuous distance function. The gap function is re-expressed allowing penalty and Lagrange multiplier formulations. The article introduces a node-to-element integration for first order elements, but also discusses signed distances, partial updates, intermediate surfaces, mortar methods and higher order elements. The algorithm is fast, simple and robust for complex geometries and self contact. The computed tractions conserve linear and angular momentum even in infeasible contact. Numerical examples illustrate the new algorithm in three dimensions.



## 1. Introduction

Many simulations involve unilateral contact/impact analysis. In dynamic simulations, notably in explicit dynamics, the number of time steps may be very large. A fast collision response is, therefore, required. This includes three components: the contact search, the generation of discrete contact conditions (constraint equations or penalty forces) and the solution of the dynamic response. Beside the numerical efficiency, accuracy of the contact algorithm is very important. It involves the geometrical accuracy of the actually measured interpenetration, the accuracy of the measured penetration depth and depth gradient, and the order of the spatial and temporal discretization scheme. This article focuses on the contact search and on the evaluation of the penetration depth and its linearization.

State-of-the-art methods are often based on a (smooth) gap function or non-smooth surface features. The gap function is a measure for the distance between a given contactor point and the target boundary surface. Many gap function algorithms rely on direct closest point projection of the contactor point onto the target surface [1,2]. These methods may become quite complex to ensure robustness of the detection. Problems may appear during the Newton iteration of the projection, when multiple candidates of contact segments appear or if the contactor point falls into a 'dead zone' (associated problems are nodes getting caught at corners and negative sliding energies in friction). These problems were first tackled by the inside-outside algorithm [3] which associates a normal vector to each contactor point that is obtained by averaging the normals of the adjacent segments. The dead-zone problem at corners and edges on the boundary is eliminated and no iterations are required. It still needs the creation of a halo, i.e. an artificial bounding volume around a surface segment, during the global search phase.

The gap function assumes smoothness of the contact boundaries. Some complex geometries, however, include non-smooth surface feature, for example corners or edges. For such problems, non-smooth contact search algorithms were developed [4–6]. Within the family of gap function algorithms, non-smooth surface features can be approached for a certain degree by replacing the boundary of the finite element mesh by a smoothed surface representation which is at least $C^1$-continuous and is typically based on splines, NURBS or NURBS-patches, see for example [7–15]. These formulations artificially smoothe the geometry of a corner such that a differentiable gap function is obtained. Further, instead of replacing the boundary of a FEM mesh by a smooth surface, one may use higher-continuous element formulations as found in isogeometric analysis [16–18] which explicitly define smooth boundaries. Boundary approximations are classified in [19] where one proposes the combination of point-to-point, point-to-edge and point-to-surface projections in order to treat $C^0$-continuous contact boundaries.

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### 1.1. Distance fields

An alternative approach to contact detection utilizing a gap function relies on the evaluation of distance fields. Distance fields provide an implicit representation of the closest point projection, see Fig. 1. The idea is that once the coordinate of a point is known, one does not compute the actual projection, but evaluates a scalar field to obtain the distance to a surface.

Algorithms regarding distance fields go back to the level set equation. The level set method was presented by Osher and Sethian [20] who described the temporal propagation of moving interfaces by numerical methods solving the Hamilton–Jacobi equation. This is performed by a finite difference scheme working on a rectangular grid in two or three dimensions. Information on normal vectors and curvature can be obtained. The fast marching method [21] provides an efficient numerical scheme of complexity $n \log n$ to compute the support values on the grid. It is a reinterpretation of the propagation process, i.e. the time where the interface passes a certain grid point is influenced only by those neighboring grid points which are previously passed by the interface. An overview on the theory of level set and fast marching methods and their applications to problems of various areas are given in [22,23], for example shape offsetting, computing distances, photolithography development, seismic travel times, etc. Distance fields are a special case of the level set equation where the absolute value of the advection velocity is 1.

The concept of distance fields was introduced to contact problems in [24] using first order tetrahedral finite elements. The distance field is generated on a supplementary grid and evaluated at the finite element nodes. Simplicity and robustness compared with closest point projection is emphasized, in particular no longer required smoothness conditions on the shape of the boundary. Self-contact, large deformations and deep interpenetrations may be treated easily. Exact intersection polygons are determined on which contact forces are computed by the penalty method [24]. More details on the employment of the distance field are provided in [25]. It focuses on the pre-computation of the distance field by fast marching. A simple partial update strategy during a time integration is proposed for regions where intersections actually occur. More details of the approach are presented in [26].

A supplementary grid is not required if the distance field is interpolated on the finite element mesh. This is constricted by the lack of efficient level set methods on unstructured meshes. A fast marching method is adopted to acute triangle meshes in [27]. The basic problem are instabilities which arise by propagating approximate levels along arbitrarily changing directions. Instead of propagating the approximate distance, [28] computes accurate distances of grid points to the initial interface, but propagates a reference to the surface patches to which the closest point projection refers to. The idea was adopted to tetrahedral finite element meshes [29] with application to collision detection eliminating the supplementary grid. A partial distance field update strategy is provided for simplex meshes therein. Although not related to distance fields, a notable partial update strategy is presented by Heidelberger et al. [30] which improves the robustness of closest point projection approximation in two dimensions. By identifying the actually intersected boundary as initial interface, the distances will only be computed with respect to the selected surface patches and the partial update is restricted to the finite element nodes which are actually in contact. The approximate distances and normal vectors are propagated similar to the original fast marching approach.

### 1.2. Objectives and outline

The objectives behind this article are to adopt the ideas behind distance fields to collision detection on unstructured grids. No supplementary grid should be used. Instead, the distance field will be interpolated using the finite element mesh, i.e. no distances outside the bodies can be represented. The discretization should be suitable for arbitrary finite element types. Further, the distance field is to be expressed in terms of the gap function which in turn enables penalty and Lagrange multiplier formulations.

Section 2 presents the basic ideas behind Eulerian distance fields and level sets on a supplementary grid and after that introduces the discretization being used in this article. The distance field is interpolated on arbitrary finite element types following the presentation in [31]. The formulation is, however, unstable. A correct distance distribution can not be represented by the formulations used in [29,31] when applied to complex geometries. This is pointed out in Section 3. Therein, a novel interpolation scheme of the distances is proposed which resolves the instabilities. Furthermore, the assumed gradient formulation of [31] will be modified to match the novel distance interpolation and to eliminate inaccuracies of the gradient field on the boundary. The presentation continues by reformulating the closest point projection through the distance field in Section 4. A simple and robust approximation to the projection will be derived from a linear expansion of the distance. Section 5 introduces some fundamentals of contact mechanics and interprets the gap function in terms of discrete distances. Numerical examples are presented in Section 6. They utilize a 'node-to-element' integration scheme on the boundaries of first order tetrahedral and hexahedral finite elements. Finally, Section 7 summarizes important properties of the new collision detection.

The appendix provides supplementary information that is required to complete the distance field algorithm. A presents Closest

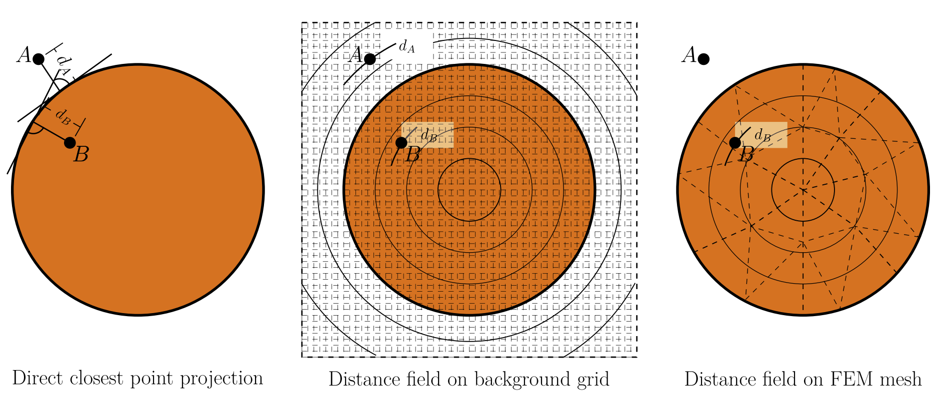


**Fig. 1.** Representations of the closest point projection of two points $A, B$ onto the boundary of a circle: Direct projection, implicit projection by distance fields interpolated on rectangular supplementary grids or interpolated on unstructured grids.