Comput. Methods Appl. Mech. Engrg. 255 (2013) 89–103

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

A goal-oriented error estimator for the analysis of simplified designs

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ARTICLE INFO

Article history:
Received 27 December 2011
Received in revised form 13 November 2012
Accepted 18 November 2012
Available online 29 November 2012

Keywords:
Defeaturing error
Analysis dependent simplification
CAD/CAE integration
Dual weighted residual

ABSTRACT

Simplifying an engineering design by removing geometric details can significantly reduce the complexity of downstream tasks of mesh generation and field solution computation. However, lack of a proper analysis and estimates of the errors induced by design simplification remains a bottleneck in seamless CAD/CAE integration. To address this issue, this paper develops a general framework for providing a posteriori estimates of goal-oriented engineering analysis error caused by removing a negative feature, or void, where material is absent, from an engineering design. Our approach is built upon the idea of representing it as a *modeling error* defined in the same geometry, which is then estimated using the dual weighted residual method. The derived error estimator still involves an uncomputable term involving dual errors, which is removed by further simplification into a term on the feature's boundary, via an exterior solution in linear cases or problem-specific approaches in nonlinear cases, utilizing classical theories of differential operators. We illustrate our approach via a semilinear second order elliptic equation, and explicit error estimators are also derived for the classical Poisson equation and linear elasticity. Results of numerical tests are shown, and comparisons made with results obtained with related approaches.

1. Introduction

Computer simulation, also referred to as engineering analysis, is typically performed using finite element analysis and related methods. It is based on a volumetric mesh derived by discretizing an engineering CAD model, the latter typically taking the form of a boundary representation (B-rep) model bounded by a set of NURBS surfaces. Converting such a CAD model into a volume mesh suitable for engineering analysis is far from trivial. It can take up to 80% of the overall analysis time for complex engineering designs according to a report from Sandia National Laboratories (see [5]).

One approach to resolving this formidable task of CAD/CAE integration is to use *isogeometric analysis* (IGA), proposed by Hughes et al. [24]. Here the same basis functions (usually NURBS, or alternatively, T-splines [5]) are used to represent the geometry *and* the solution space of dependent variables. The advantages of IGA for analysis are numerous: for example, exact geometry representation, simple mesh generation and refinement, and direct communication with CAD geometries [5,13,24,38,43]. Generating an IGA mesh for a single NURBS patch is straightforward, while doing so for a trimmed NURBS patch can generally be handled using T-splines [5,31,40] or trimming techniques [27]. Creating an IGA solid mesh for general 3D volumes is much more challenging, and recent progress toward this topic is referred to the work of Zhang et al. [48,50,51] or Xu et al. [49].

As well as mesh generation, CAD/CAE integration also involves a very time consuming and complex process called *geometry preparation* or *idealization*, which creates an analysis-suitable geometry via steps of dimension reduction [2,28,32] and geometry simplification. It typically requires much user interaction, and accounts for 57% of the overall analysis time at Sandia [5]; in comparison, mesh generation only accounts for 23% of the overall time. A major part of geometry simplification is *defeaturing*, removing geometric details such as holes, fillets, blends, and slots, from a complex CAD model. Doing so significantly reduces the time needed both for meshing and field solution computation performed on the mesh, and avoids potential mesh generation failure or ill-conditioned computations that may produce inaccurate analysis results. Consider for example, Fig. 1. The number of mesh elements used to represent the original geometry is much higher than for the simplified geometry (using the same meshing parameter settings), due to the requirements that the mesh should be adapted to the geometry, while having a smooth transition in element size.

The benefits of geometric simplification, however, come at the cost of (hopefully small) differences in the results of the analysis. Understanding the engineering analysis error induced by defeaturing, i.e., the *defeaturing error*, is essential if we are to ensure that a desired analysis accuracy can still be met after geometric simplification. Providing error estimates is necessary for both traditional finite element analysis and for the more novel IGA approach, as

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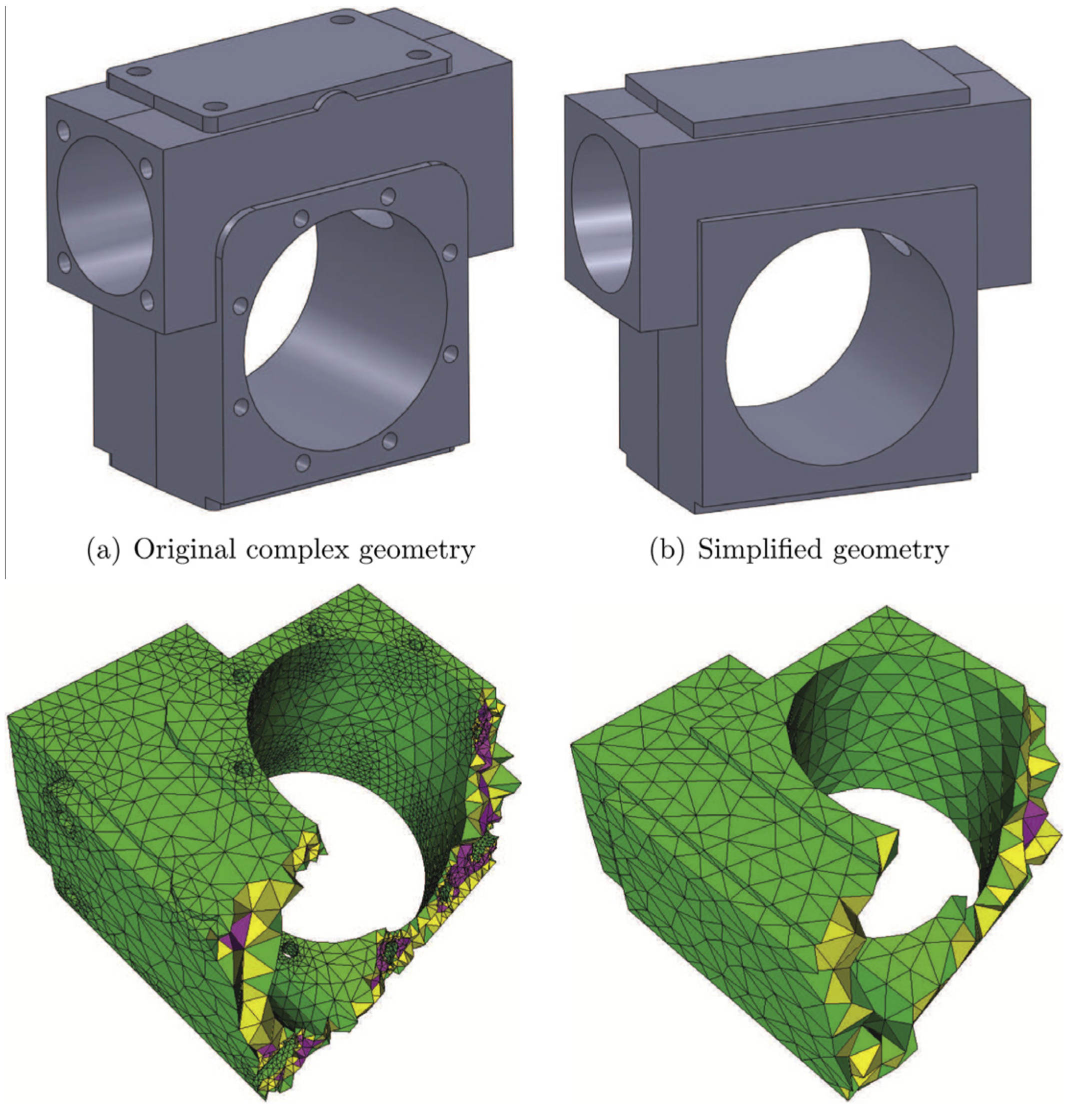
http://dx.doi.org/10.1016/j.cma.2012.11.010



(a) Original complex geometry (b) Simplified geometry

(c) Mesh for (a) with 86030 elements (d) Mesh for (b) with 3990 elements

Fig. 1. Suppressing geometric details (36 holes, 8 blends and 2 extrusions) from the original complex geometry in (a) gives the simplified geometry in (b). Many fewer mesh elements are produced for the simplified geometry than the original geometry, given the same meshing parameters.

geometry simplification and idealization are still unavoidable for complex CAD models, as noted by Hughes et al. [24].

A full treatment of geometric simplification for engineering analysis is an extremely difficult task. This paper focuses on one of the core issues—providing proper estimates of changes in specific quantities of engineering interest due to the removal of a *negative feature*, or void, where material is absent, from an originally complex geometry. This *goal-oriented* error is estimated in an a posteriori sense by using analysis solutions for the simplified geometry but not solutions for the original complex geometry.

We give a general framework for a posteriori estimates of goal-oriented defeaturing errors for a broad class of linear and nonlinear physical phenomena. Our approach utilizes the *dual weighted residual* (DWR) method, originally developed by Becker and Rannacher [7] for finite element approximation error estimates, and extended by Oden and Prudhomme [35] to estimate modeling errors. By taking the solution for the simplified geometry as the solution restricted to the original geometry (as a sub-region of the simplified geometry), the defeaturing error is converted into a modeling error over the same geometry, caused by applying different boundary conditions over the internal boundary. This reformulated error can be then estimated using the DWR method, but this estimate still involves an uncomputable term of dual errors containing solutions for the original geometry. We thus further simplify the derived error estimate into a local quantity on the boundary of the negative feature. This simplification allows the dual error term to be estimated via an exterior solution in linear cases, or by utilizing classical theories of differential operators in nonlinear cases, ultimately resulting in an error estimate defined on the negative feature's boundary. We illustrate our overall approach using a semilinear second order elliptic equation, as well as deriving error estimates for the classical Poisson equation and linear elasticity. We also validate our results with numerical tests.

The idea of converting defeaturing error into modeling error was first described in [29] for linear elasticity, and extended in [30] for general nonlinear problems. However, both approaches are theoretically unjustified as they simply discard the uncomputable terms of the dual errors without attempting to estimate them. Unlike these approaches, here we estimate the dual error terms, using separate approaches for linear and nonlinear cases as noted above. As a result, our new error estimators (called *defeaturing error estimators* (DEEs)) are more widely applicable, and further