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A finite element scheme with the aid of a new carving technique combined with smoothed integration

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ABSTRACT

An efficient scheme is proposed that can generate appropriate meshes for three-dimensional complex domains, leading to accurate solutions in finite element analysis. After preparing a structured mesh with hexahedral elements, the surface information of an arbitrary domain is soaked into the mesh. The elements around the surface are trimmed according to the surface information, and then they are replaced by polyhedral elements while the interior of the domain remains occupied by the original hexahedral elements. The scheme is further extended to manage complex domains with discontinuities in geometry and material properties such as a polycrystalline structure comprised of multiple materials or grains, as well as highly curved domains with a single material. Subsequently, the generated mesh, composed of the polyhedral elements on the surface and the hexahedral elements in the interior, is directly applied for finite element analysis in conjunction with smoothed integration in the strain smoothing technique. Because the smoothed integration does not require the shape function derivatives, the efficient analysis can be conducted without explicitly defined shape functions of the polyhedral elements. The smoothed integration can improve the accuracy of the numerical solutions as well. The present study focuses on reporting a new idea on a finite element scheme using the polyhedral elements with the smoothed integration, rather than providing a general and systematic approach; however, it has a great potential to deal with general and practical problems. Through numerical examples, it is shown that the proposed scheme provides accurate solutions in a simple and efficient manner.

1. Introduction

In finite element method (FEM), it is well known that hexahedral elements are preferred to tetrahedral elements due to their superior performance. However, it is difficult to generate a mesh with only hexahedral elements for domains with complex geometries. For this reason, tetrahedral elements are widely used for three-dimensional mesh generation due to their adaptability for arbitrary configurations. Although many acceptable methods for automatically generating the hexahedral meshes have been proposed [1–3] including grid-based methods [4–7] and advancing front methods [8,9], hexahedral mesh generation remains a challenge from the viewpoint of efficiency and versatility for very complicated domains, in addition to the quality of the generated meshes [10–12].

Recently, Sohn et al. [13] proposed a scheme, termed the *carving technique*, that can generate meshes for complex structures based on the hexahedral elements. A basic concept of this scheme is borrowed from the grid-based methods. The carving technique is divided into two processes: trimming and splitting processes. In the trimming process, the surface information of a given domain is first inserted into a structured hexahedral mesh that is used as a background grid over the domain. Subsequently, the hexahedral elements intersecting the domain surface are trimmed using the marching cube algorithm [14,15]; thus, they are converted to *polyhedral elements* with trimmed faces. In the splitting process, the polyhedral elements are decomposed into several *poly-pyramid elements* of which the shape functions are derived using a moving least square approximation [16–18]. It was demonstrated that the carving technique could successfully provide conforming meshes that lead to accurate solutions in finite element (FE) analysis [13].

In this paper, the carving technique is further improved in terms of efficiency and accuracy. To eliminate the splitting process in which the poly-pyramid elements are used, the polyhedral elements are used directly for FE analysis. Furthermore, only the elements around the surface are replaced by the polyhedral elements, while the interior of the domain is occupied by the

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remaining hexahedral elements. The procedure of the mesh generation is thus simplified without spatial repartition of the polyhedral elements. In finite volume method (FVM), the polyhedral elements have already been used to enhance the flexibility of the mesh generation and accuracy of the solution: it is known that they are more flexible in meshing and more accurate than the tetrahedral and hexahedral elements due to the fact that they have more neighboring elements or cells than the tetrahedral and hexahedral elements [19–22]. In the conventional FVM, the shape functions do not need to be calculated over the entire element, whereas it is necessary, albeit difficult, to define the shape functions of the polyhedral elements for the use in FEM. Their shape functions are generally given in the form of rational functions, not polynomial functions, similar to those of polygonal elements in two-dimensional domains [23,24]. Not only do the rational functions involve a complicated formulation, but they also cause numerical errors with Gauss integration. For this reason, the polyhedral elements have not been applied in FE analysis with a few exceptions [25–28].

In order to resolve such difficulties in the formulation and integration of the shape functions, polyhedral elements with smoothed integration are proposed in this paper by borrowing the concept from the cell-based smoothed finite element method (CSFEM) [29–37]. Thus far, the smoothed integration has not been applied for arbitrary polyhedral elements. The smoothed integration in CSFEM is based on the stabilized conforming nodal integration in the meshless methods [38] and it leads to some important merits over the Gauss integration in the conventional FEM. Because the smoothed integration does not require shape function derivatives, the FE analysis can be conducted without explicitly defined shape functions for the polyhedral elements [31–35]. The smoothed integration also allows severe distortion of the elements because the physical-to-parental mapping is not required [29–37]. Accordingly, the smoothed integration simplifies the procedure of calculating a stiffness matrix and also removes the numerical errors arising from the rational types of shape functions. In two-dimensional cases, it has been reported that the patch tests with smoothed integration are passed using nonconventional types of elements, such as the polygon elements [34] and variable-node elements that contain an arbitrary number of nodes on each edge of the quadrilateral element [35].

Hereafter, CTPPG indicates the carving technique with the previous poly-pyramid elements and Gauss integration [13], and CTPHS is an improved version of the carving technique, to be presented in this paper, with the polyhedral elements and the smoothed integration without the splitting process. Furthermore, in this paper, it is shown that the CTPHS enables one to manage domains with discontinuities in material property or geometry, such as a polycrystalline structure comprised of multiple materials or grains, as well as domains with a single closed surface. In Table 1, the present study is compared with the previous study [13].

The remainder of this paper is organized as follows. In Section 2, the mesh generation through the trimming process is described with the special treatment for the domains with multiple materials or grains; the applications of the smoothed integration to the polyhedral elements are considered in Section 3. Then, in order to examine the effectiveness of the CTPHS in terms of efficiency and accuracy, numerical examples are provided in Section 4. Finally, some concluding remarks are presented in Section 5.

2. Mesh generation with hexahedral and polyhedral elements

In this section, the mesh generation procedure is described with an emphasis on the differences between the CTPPG and the CTPHS. Subsequently, an additional scheme is presented in order to manage domains with multiple materials or grains.

2.1. Construction of polyhedral elements by trimming hexahedral elements

As mentioned above, the CTPPG is divided into trimming and splitting processes [13]. In the trimming process, a structured background mesh with the hexahedral elements is first constructed to overlay a specified domain whose surface information is given. In this study, the surface information is given in the form of stereolithography (STL) files from computer-aided design tools. In the STL files, outer surfaces of a model are approximated using many small triangles enough to smoothly represent the surfaces. Through soaking the surface information into the background mesh, the hexahedral elements are trimmed as shown in Fig. 1(a). The marching cube algorithm [14,15] classifies the eight nodes of each hexahedral element into active and non-active nodes, and identifies the intersection topology by determining which edges intersect with the specified surface. The active nodes enclosed by and located on the surface are included in the model, while the non-active nodes are excluded in the next process. According to the intersection topologies shown in Fig. 1(a), the polyhedral elements are generated around the surface. The trimmed faces are triangulated because they may not be coplanar. Subsequently, in the splitting process, the trimmed element is reconstructed by introducing a new node, indicated by the red solid circle in Fig. 1(b), inside the element. By connecting the polygon-shaped faces to the added node, the polyhedral element is decomposed into several poly-pyramid elements, as shown in Fig. 1(b). Each of the poly-pyramid elements is an individual element, and these are used in calculating the element stiffness matrix with Gauss integration in the same manner as the conventional elements, e.g. the tetrahedral and hexahedral elements. In Fig. 1(b), the poly-pyramid elements that contain the triangular surfaces resulting from the trimming process are placed in the original hexahedral elements indicated by broken red lines for clarity.

In this paper, the mesh generation procedure is further simplified by eliminating the splitting process described in Fig. 1(b). That is, the CTPHS, proposed in this paper, provides meshes with polyhedral elements directly from the trimming process, without the splitting process, and therefore greatly enhances the efficiency in the mesh generation procedure. Consequently, the polyhedral elements generated from the trimming process are used around the domain surface while the hexahedral elements from the initial structured mesh occupy the inner domain. In the CTPHS, the smoothed integration makes it possible for the meshes with polyhedral elements to be applied efficiently in FE analysis, as will be discussed in Section 3.

Table 1
Differences between the present and previous [13] studies.

Differences	Present study	Previous study [13]
Modeling processes	Trimming process	Trimming and splitting processes
Element form	Polyhedral element	Poly-pyramid element
Integration form	Surface integration with strain smoothing technique	Volume integration with Gauss integration
Integration unit	Smoothing cell	Gauss integration point
Shape function	Implicitly defined values at vertices of smoothing cells	Explicitly defined values over poly-pyramid elements by moving least square approximation
Applicable domains	Single and multiple domains	Single domains