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On the lack of rotational equilibrium in cohesive zone elements

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ABSTRACT

This paper unravels an intrinsic shortcoming of several cohesive zone models, whereby commonly adopted traction separation laws do not necessarily satisfy rotational equilibrium at the cohesive element level. The necessary condition to ensure rotational equilibrium is derived. To demonstrate the error that is caused by the lack of rotational equilibrium, examples for three traction separation laws are shown, for both homogeneous and inhomogeneous deformation. For a large range of values of the critical opening length the error scales almost linearly with this critical opening length. For large critical opening length the error may even become significant, requiring the use of a traction separation law that satisfies the condition for rotational equilibrium.

1. Introduction

Since the early work by Dugdale on ductile materials [1] and on quasi-brittle materials by Barenblatt [2], cohesive zone models have been used extensively to describe failure in a wide variety of materials and interfacial systems, see e.g. [3–5]. Research has been done on e.g. metallic materials [6], composite materials [7–9], and failure in polymer/metal interfaces [10–14], possibly involving large deformations in the failure zone.

In literature much attention has been given to different aspects of cohesive zone modeling. For example, the shape of the traction-separation law [15,16], influence of mode-mixity [8], numerical issues [17–20], large deformations [12] and thermodynamical consistency [21,22] have been studied. However, one important point that has not yet been addressed, concerns the rotational equilibrium when using cohesive zone models.

Cohesive zone formulations are based on a traction continuity condition. Whilst this approach naturally satisfies the balance of forces, the rotational equilibrium is not necessarily satisfied, as demonstrated in this paper.

The requirement on the traction-separation law in order to satisfy rotational equilibrium at the structural level is derived. Demonstration problems show the influence of the lack of rotational equilibrium on the obtained results in the case of uniform and non-uniform deformation. Especially when the critical opening length is large, the lack of rotational equilibrium may be a real concern for practical applications. Examples of such applications, where the rotational equilibrium might require some detailed consideration, can be found in e.g. [11,23,24]. Cohesive zone laws that do not suffer from this intrinsic shortcoming are also identified.

2. Problem statement

In cohesive zone models, the material behavior is described by a traction-opening relation instead of the classical stress–strain relation used in a continuum. Rotational equilibrium in a non-polar continuum is equivalent with the requirement that the Cauchy stress tensor is symmetric. For cohesive zone models this can not be enforced as the full stress tensor is not available. The only equilibrium requirement in cohesive zone models is continuity of tractions. Note that, for large deformations, this should be a force equilibrium and not a traction equilibrium [12]. However, as demonstrated in this section, traction (or force) equilibrium is not a sufficient condition to obtain rotational equilibrium as well, i.e. the sum of all moments does not necessarily vanish in the deformed configuration.

2.1. Demonstration problem

The demonstration problem is shown in Fig. 1. It consists of a single planar cohesive zone with length L and unit out-of-plane thickness. Initially the top and bottom plane coincide. It should be emphasized that the length L relates to the structural length, not to the element size. Later, the influence of the structural length L is addressed and it also shown that the analysis does not depend on the element size.

During deformation the top and bottom plane remain parallel and of the same length, i.e. the cohesive zone element deforms homogeneously. The displacement of each point on the top plane is described by the vector $\vec{\Delta} = \vec{\Delta}_n + \vec{\Delta}_t$, where (n) and (t) indicate

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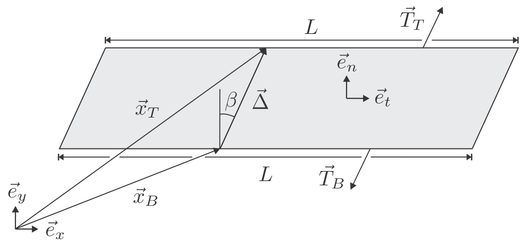

Fig. 1. Homogeneously deformed cohesive zone element.

normal and tangential direction respectively. The angle of the opening vector with respect to the y-axis is denoted by β.

The moment with respect to an arbitrary point in space is calculated as

$$\vec{M} = \vec{M}_T + \vec{M}_B \tag{1}$$

where

$$\vec{M}_T = \int_0^L \vec{x}_T \times \vec{T}_T \, dx \tag{2}$$

$$\vec{M}_B = \int_0^L \vec{x}_B \times \vec{T}_B \, dx \tag{3}$$

In this expression $\vec{T}_T$ and $\vec{T}_B$ are the Cauchy tractions acting on the top and bottom planes of the cohesive zone respectively and $\times$ is the cross product. The tractions are typically determined through the use of a so-called traction-separation law (TSL), relating the traction to the opening of the cohesive zone. It is common practice to decompose the traction into its normal (n) and tangential (t) components.

2.2. TSL condition for rotational equilibrium

From the expression for the total moment, Eq. (1), a requirement on the traction-separation law to obtain rotational equilibrium can be easily derived. Using $\vec{x}_T = \vec{x}_B + \vec{\Delta}$, and traction continuity ($\vec{T} \equiv \vec{T}_T = -\vec{T}_B$), Eq. 1 can be written as

$$\vec{M} = \vec{M}_T + \vec{M}_B \tag{4}$$

$$= \int_0^L (\vec{x}_T \times \vec{T} - \vec{x}_B \times \vec{T}) \, dx \tag{5}$$

$$= \int_0^L (\vec{\Delta} \times \vec{T}) \, dx \tag{6}$$

and since, in this part, a homogeneous deformation state is considered the integrand is constant and the moment only vanishes when

$$\frac{\Delta_t}{\Delta_n} = \frac{T_t}{T_n} \tag{7}$$

It can thus be concluded that the traction vector should be aligned with the opening vector to obtain a zero residual moment. In Section 5 it is shown that this condition also yields rotational equilibrium in the case of non-homogeneous deformation.

It should be noted that such aligned traction vectors also exist in some specific TSL, e.g. [12,25]. However, to the authors' knowledge, the relation between rotational equilibrium and alignment of TSL has not been reported or exploited before. Note that the traction-opening alignment of the TSL does not imply that mode dependency can not be accounted for, as was shown for example in [26].

2.3. Residual moment

For aligned TSLs the residual moment automatically vanishes. However, for non-aligned TSLs there is a residual moment, as illustrated here. For demonstration purposes, three different TSLs are next considered of which the decoupled responses are shown in Fig. 2, i.e. $T_n(\Delta_n, \Delta_t = 0)$ (left pictures) and $T_t(\Delta_t, \Delta_n = 0)$ (right pictures). The expressions for these traction-opening relations are listed in Appendix A. In the Tvergaard [27] model, α determines the ratio of the maximum tangential traction to the maximum normal traction: $\alpha = T_{t,max}/T_{n,max}$. It is important to note the different definitions of the δ parameters. For the Tvergaard and Geubelle & Baylor [28] models these parameters give the opening at which complete failure occurs whereas for the improved Xu-Needleman [29] these are related to the opening at which the traction reaches its maximum value.

The moment, as determined from Eq. (1), is plotted in Fig. 3 as a function of the effective opening length $\Delta = \sqrt{\Delta_n^2 + \Delta_t^2}$ for different values of the loading angle β (see Fig. 1). The cohesive zone parameters are listed in Table 1 and the length L is taken equal to 1 mm. From Fig. 3 it becomes clear that the residual moment is not zero, except in pure mode I or mode II loading, corresponding to $\beta = 0$ and $\beta = \pi/2$, respectively.

3. Error formulation

In the previous sections it was shown that a cohesive zone element is only in rotational equilibrium if the traction is aligned with the opening. However, cohesive zone models employing non-aligned traction-separation laws have been used extensively in the literature for a wide range of applications. To assess the accuracy of these models it is necessary to qualify and quantify the consequence of this deficiency.

3.1. Equivalent continuum

For the analysis, a continuum model is used that reveals the same traction-opening behavior as the cohesive zone model. Because of the symmetry of the stress tensor the continuum remains in rotational equilibrium. Note that the connection between a regular continuum and a cohesive zone model is not new (e.g. [30]),

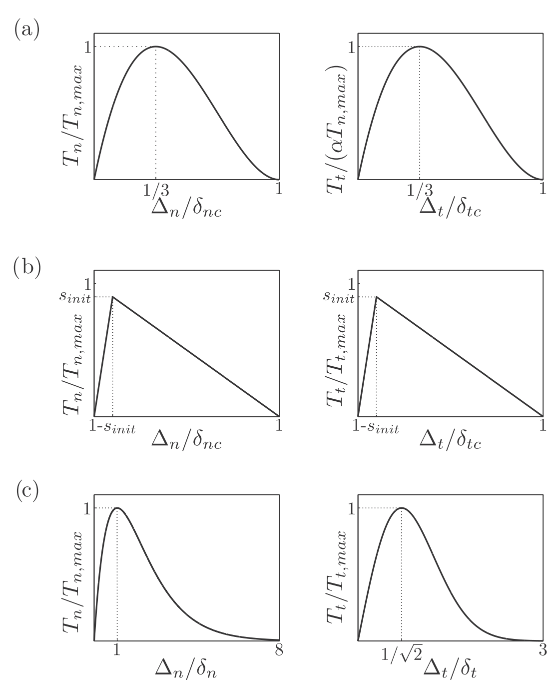

Fig. 2. Decoupled response of traction separation laws used in this study; (a) quadratic, Tvergaard (T) (b) bi-linear, Geubelle & Baylor (G & B) (c) exponential, improved Xu-Needleman (I X-N).