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# Unsteady stagnation point flow of viscous fluid caused by an impulsively rotating disk

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ABSTRACT

This paper describes the influence of mass transfer with first order chemical reaction and concentration buoyancy effect on the unsteady magnetohydrodynamic (MHD) stagnation point flow of a viscous fluid. The governing partial differential equations are reduced into the ordinary differential equations by using similarity transformations. Homotopy analysis method (HAM) is employed for the computations of solution. The effects of some interesting parameters on the dimensionless radial and axial velocities and dimensionless concentration field are examined. The variations of skin friction coefficients and Sherwood number are analyzed through plots.



## 1. Introduction

The study of flow due to rotating disk is quite popular because of its applications in many engineering processes. For instance, spin-coating, manufacturing and use of computer disks, rotational viscometer, centrifugal machinery, pumping of liquid metals at high melting point, crystal growth from molten silicon and turbomachinery etc. In view of both industrial and technological applications, considerable attention has been paid to the flow due to rotating plates and disks. Karman (1921) was first to construct similarity solution of viscous fluid flow due to rotating disk. Cochran (1934) discussed asymptotic solution for the steady MHD flow caused by a rotating disk through Von Karman transformation. The influence of suction/injection on the flow driven by rotating disk has been studies by Stuart (1954). Benton (1966) subsequently extended the work of Cochran to the unsteady flow due to impulsive rotation of disk. McLeod (1969) studied the existence of the solution of swirling flow. Kuiken (1971) studied the flow near a rotating disk subjected to normal blowing. Effects of magnetic field on the flow have been considered by the number of researchers, for instance, (Ariel, 2001; Watanabe and Oyama, 1992). Zandbergen and Dijkstra (1987) investigated the flow between two rotating disks. Ersoy (1999) considered MHD flow of an Oldroyd-B fluid between eccentric rotating disks. Miklavcic and Wang (2004) analyzed the swirling flow caused by rotating disk taking partial slip into account. Asghar *et al.* (2007) studied non-Newtonian flow due to non-coaxial rotation of an accelerated disk and fluid at infinity. The effect of heat transfer in such flow problems has been also discussed. For instance, Sparrow and Gregg (1959) analyzed the effect of Prandtl number on the temperature. Sparrow and Cess (1962) considered heat transfer characteristics on the flow by a rotating disk. Kumar *et al.* (1988) analyzed the effect of heat transfer on the flow due to rotating porous disk. Maleque and Sattar (2005) studied convective flow on a porous rotating disk. They considered the variable fluid properties. Wiesche (2007) examined the heat transfer from rotating disk in a parallel air cross flow. Turkyilmazoglu (2009) computed exact solution for the flow of heat transfer in an incompressible viscous fluid over a porous disk. He noticed the effects of Prandtl and Eckert numbers on the dimensionless temperature. The mass transfer effect in such flows has been also studies. Very recently, Osalusi *et al.* (2008) has considered the Dufour and Soret effects on MHD convective flow by a rotating disk with slip effects. After this, Maleque (2000) conducted an analysis regarding the Dufour and Soret effects on unsteady convective heat and mass transfer on MHD flow due to a rotating disk. Dinarvand (in press) derived series solution for stagnation point flow towards a rotating disk by using HAM. Dinarvand *et al.* (2010) constructed series solutions for MHD unsteady flow near

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**Nomenclature**

| | |
|---|---|
| $a$ | stretching rate |
| $\mathbf{B}$ | applied magnetic field $[0, 0, B_0]$ |
| $C$ | concentration function |
| $C_F$, $C_G$ | skin friction coefficients in radial and azimuthal direction respectively |
| $D$ | diffusion coefficient of diffusing species |
| $F$ | dimensionless radial velocity |
| $G$ | dimensionless azimuthal velocity |
| $\bar{g}$ | gravitational acceleration |
| $Gc$ | local mass Grashof number |
| $\mathbf{J}$ | Current density vector |
| $K_1$ | chemical reaction constant |
| $M$ | Hartman number |
| $r, \theta, z$ | cylindrical coordinates |
| $Sc$ | Schmidt number |
| $Sh$ | Sherwood number |
| $u, v, w$ | velocity components |
| $u_e$ | free stream velocity |
| $\mathbf{V}$ | velocity field |

*Greeks symbols*

| | |
|---|---|
| $\beta_c$ | coefficient of expansion due to concentration difference |
| $\gamma$ | first order chemical reaction parameter |
| $\eta$ | dimensionless space variable |
| $\lambda$ | rotation parameter |
| $\Omega$ | constant angular velocity |
| $\nu$ | kinematic viscosity |
| $\rho$ | fluid density |
| $\sigma$ | electrical conductivity of the fluid |
| $\tau$ | dimensionless time |

forward stagnation point of an impulsively rotating and translating sphere in the presence of the buoyancy forces. Literature survey reveals that most of the studies are restricted to heat transfer characteristics on flow induced by the rotating bodies. However, investigation regarding the effect of mass transfer on unsteady stagnation point flow induced by disk suddenly set into rotation are fewer.

The main objective of present work is to discuss unsteady stagnation point flow caused by an impulsive rotation of disk. Mass transfer with first order chemical reaction is considered. The concentration buoyancy effects are included. The governing partial differential equations are transformed into the ordinary differential equations by using similarity transformation. The resulting non-linear coupled problems are solved analytically by employing homotopy analysis method (HAM). Homotopy analysis method is powerful technique which has been already employed by many researchers (Abbasbandy, 2008; Abbasbandy and Parkes, 2008; Abbasbandy and Zakaria, 2008; Allan, 2007; Cheng and Liao, 2008; Dinarvand, in press; Dinarvand and Rashidi, 2010; Dinarvand *et al.*, 2010; Hashim *et al.*, 2009; Hayat and Nawaz, 2009; Hayat *et al.*, 2009, 2010; Liao, 2003, 2004, 2005) for the solutions to different complicated problems. The outline of this paper is as follows. In section two, the mathematical formulation, definition of skin friction coefficients and Sherwood number are presented. Section three extends the application of HAM to construct series solutions of the governing nonlinear system. In section four, the results and discussion are given. The conclusions are summarized in section five.

## 2. Mathematical formulation

Consider unsteady axisymmetric flow of an electrically conducting viscous fluid bounded by an infinite disk at $z = 0$. Initially the disk is at rest. The disk is impulsively set into rotation about $z$-axis with constant angular velocity $\Omega$. A uniform magnetic field $\mathbf{B}_o$ perpendicular (parallel to $z$-axis) to the plane of disk is applied and concentration buoyancy effect is considered. There is no applied electric field and induced magnetic field is neglected under the assumption of small magnetic Reynolds number. In addition mass transfer with first order chemical reaction occurring in the flow regime is taken into account. The concentration field at disk as well as at free stream velocity is taken constant. The physical model and coordinates system for the considered problem is show in Fig. 1. The fundamental laws that are helpful for the description of present problems are

$$\nabla.\mathbf{V} = 0, \tag{1}$$

$$\rho\frac{d\mathbf{V}}{dt} = \mu\nabla^2\mathbf{V} + \rho\beta_c\mathbf{g}(C - C_\infty) + \mathbf{J}\times\mathbf{B}, \tag{2}$$

$$\frac{dC}{dt} = D\nabla^2 C - K_1(C - C_\infty), \tag{3}$$

$$\mathbf{J} = \sigma[\mathbf{V}\times\mathbf{B}] \tag{4}$$

in which $d/dt$ is the material derivative, $\mathbf{V}$ is the velocity vector, $\mu$ is the viscosity of the fluid, $\rho$ is the fluid density, $D$ is the diffusion coefficient, $C$ is the concentration field, $C_\infty$ is the concentration at free stream, $\bar{\mathbf{g}}$ is the gravitational acceleration, $\beta_c$ is the coefficient of expansion due to concentration, $\mathbf{J}$ is the current density vector, $\mathbf{B}(=[0,\ 0,\ B_0])$ magnetic induction vector, $\sigma$ is the electrical conductivity of the fluid, $K_1$ is the first order chemical reaction constant and electric and induced magnetic fields are neglected. The velocity and concentration fields for axisymmetric flow are of the following form

$$\mathbf{V} = [u(r,z), v(r,z), w(r,z)], \quad C = C(r,z) \tag{5}$$

Using above definitions of velocity and concentration fields in Eqs. (1)–(4), we have

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{6}$$

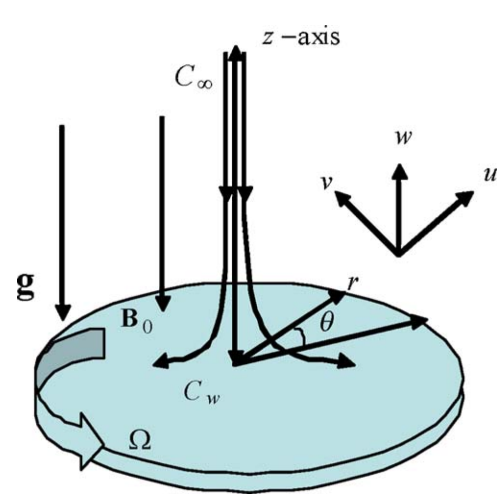


**Fig. 1.** Sketch of the physical model and coordinate system.