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Influence of heat and chemical reactions on Walter's B fluid model for blood flow through a tapered artery

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ABSTRACT

The effects of heat transfer on the blood flow of a Walter's B fluid through a tapered artery with a mass transfer under the assumption of mild stenosis have been studied. The problem is model in cylindrical coordinates system. The analytical solutions have been carried out using regular perturbation method by taking α as perturbation parameter. The expressions for velocity, temperature, concentration, resistance impedance, wall shear stress and shearing stress at the stenosis throat have been evaluated. The graphical results of different type of tapered arteries (*i.e.* converging tapering, diverging tapering, non-tapered artery) have been examined for different parameters of interest. Trapping phenomena have been discussed at the end of the article.

1. Introduction

The investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis (Nagarani and Sarojamma, 2008). Vast amount of studies have been made to study blood flow through arteries (Mekheimer and Et Kot, 2008; Sankar and Hemalatha, 2006; Sankar and Lee, 2009). Recently, heat transfer analysis have been received the attention (Nadeem and Akbar, 2009; Srinivas and Gayathri, 2009; Srinivas and Kothandapani, 2008; Srinivas *et al.*, 2009) due to its large number of applications in processes like hemodialysis and oxygenation. Bioheat is currently considered as heat transfer in the human body. In view of this thermotherapy and the human thermoregulation system (Srinivas and Kothandapani, 2008), the model of bioheat transfer in tissues has been attracted by the biomedical engineers. In fact the heat transfer in human tissues involves complicated processes such as heat conduction in tissues, heat transfer due to perfusion of the arterial–venous blood through the pores of the tissue, metabolic heat generation and external interactions such as electromagnetic radiation emitted from cell phones. In the recent past, the study of the combined effects of heat and mass transfer on biofluids has become quite interesting to many researchers both from the theoretical and experimental or clinical point of view (Chakravarty and Sen, 2005; Eldabe *et al.*, 2007; Nadeem and Akbar, 2009). Most of the studies concentrated only on the heat transfer (Khanafer *et al.*, 2007; Ogulu and Abbey, 2005) to blood flowing through the arteries but limited attentions have been focused to study the mass transport processes (Kawase and Ulbrecht, 1983; Valencia and Villanueva, 2006). There might be so many reasons on tackling blood flow in problems the presence of mass transfer. One major problem is highlighted by (Friedman and Ehrlich, 1975) that the problems of mass transport are highly convection dominated because of the low diffusion coefficients of the principal constituents govern.

In view of the above analysis, the aim of the present article is to discuss the heat and mass transfer effects on the blood flow of a Walter's B fluid (Baris, 2002) through a tapered artery with a stenosis. The governing equations are solved analytically by regular perturbation method. The expression for velocity, temperature, concentration, resistance impedance, wall shear stress and shearing stress at the stenosis throat have been calculated. At the end, the physical features of various emerging parameters have been discussed by plotting the graphs. Trapping phenomena have been discussed at the end of the article.

2. Mathematical formulation

Let us consider the flow of an incompressible Walter's B fluid lying in a tube having length L. We are considering cylindrical coordinates system $\left(\bar{r}, \bar{\theta}, \bar{z}\right)$ such that $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the velocity

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Nomenclature

B_r	Brickmann number
c_p	specific heat
D	coefficients of mass diffusivity
k	thermal conductivity
K_T	thermal-diffusion ratio
M	Hartmann number
n	stenosis shape
Q	flow rate
S_c	Schmidt number
S_r	Soret number
$\bar{T}$	temperature
T_m	temperature of the medium
u	velocity component in r-direction
w	velocity component in z-direction
Greek symbols	
α	Walter's B fluid parameter
δ	height of the stenosis
μ	Kinmatic viscosity
ν	Kinematic viscosity
ρ	Density of the fluid
ϕ	tapered angle

components in $\bar{r}$, $\bar{\theta}$ and $\bar{z}$ direction respectively. Heat and mass transfer phenomena are taken into account by giving temperature $\bar{T}_1$ and concentration $\bar{C}_1$ to the wall of the tube, while at the centre of the tube we are considering symmetry condition on both temperature and concentration. Followed by Mekheimer and Et Kot (2008), the geometry of the stenosis which is assumed to be symmetric is defined as

$$h(z) = d(z)[1 - \eta_1(b^{n-1}(z-a) - (z-a)^n)],\quad a \le z \le a+b,\quad a = d(z),\ \text{otherwise} \tag{1}$$

with

$$d(z) = d_0 + \xi z, \tag{2}$$

where $d(z)$ is the radius of the tapered arterial segment in the stenotic region, d_0 is the radius of the non-tapered artery in the non-stenoic region, ξ is the tapering parameter, b is the length of stenosis, $(n \ge 2)$ is a parameter determining the shape of the constriction profile and referred to as the shape parameter (the symmetric stenosis occurs for $n = 2$) and a indicates its location as shown in Fig. 1. The parameter η is defined as

$$\eta = \frac{\delta^* n^{\frac{n}{n-1}}}{d_0 b^n (n-1)}, \tag{3}$$

The flow equations in the presence of heat and mass transfer are defined as

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{4}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\boldsymbol{\tau}}_{\bar{r}\bar{r}}) + \frac{\partial}{\partial \bar{z}}(\bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}) - \frac{\bar{\boldsymbol{\tau}}_{\bar{\theta}\bar{\theta}}}{\bar{r}}, \tag{5}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{w} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}) + \frac{\partial}{\partial \bar{z}}(\bar{\boldsymbol{\tau}}_{\bar{z}\bar{z}}), \tag{6}$$

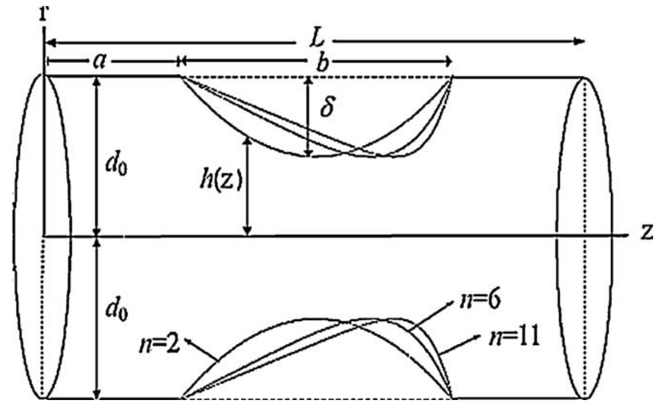

Fig. 1. Geometry of an axially nonsymmetrical stenosis in the artery.

$$\rho c_p\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{T} = \bar{\boldsymbol{\tau}}_{\bar{r}\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}} + \bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}\frac{\partial \bar{u}}{\partial \bar{z}} + \bar{\boldsymbol{\tau}}_{\bar{z}\bar{z}}\frac{\partial \bar{w}}{\partial \bar{z}} + k\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right), \tag{7}$$

$$\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{C} = D\left(\frac{\partial^2 \bar{C}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right) + \frac{DK_T}{T_m} \times \left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right). \tag{8}$$

In the above equations, $\bar{p}$ is the pressure $\bar{u}$, $\bar{w}$ are the respective velocity components in the radial and axial directions respectively, $\bar{T}$ is the temperature, $\bar{C}$ is the concentration of fluid, ρ is the density, k denotes the thermal conductivity, c_p is the specific heat at constant pressure, T_m is the temperature of the medium, D is the coefficients of mass diffusivity, K_T is the thermal-diffusion ratio.

The constitutive equation for Walter's B fluid is given by Baris (2002)

$$\bar{\boldsymbol{\tau}} = 2\eta_0 \mathbf{e} - 2k_0\frac{\delta \mathbf{e}}{\delta t}, \tag{9a}$$

$$\mathbf{e} = \boldsymbol{\varpi}\mathbf{V} + (\boldsymbol{\varpi}\mathbf{V})^{\mathrm{T}}, \tag{9b}$$

$$\frac{\delta \mathbf{e}}{\delta t} = \frac{\partial \mathbf{e}}{\partial t} + \mathbf{V}.\nabla \mathbf{e} - \mathbf{e}\boldsymbol{\varpi}\mathbf{V} - (\boldsymbol{\varpi}\mathbf{V})^{\mathrm{T}}\mathbf{e}, \tag{9c}$$

in which $\bar{\boldsymbol{\tau}}$ is the extra stress tensor, η_0 is the coefficient of viscosity, $\mathbf{e}$ is the rate of strain tensor and $\delta/\delta t$ denotes the convected differentiation of a tensor quantity in relation to the material motion.

We introduce the non-dimensional variables

$$\begin{aligned}
&r = \frac{\bar{r}}{d_0},\quad z = \frac{\bar{z}}{b},\quad w = \frac{\bar{w}}{u_0},\quad u = \frac{b\bar{u}}{u_0\delta},\quad p = \frac{d_0^2\bar{p}}{u_0 b\eta_0},\quad h = \frac{\bar{h}}{d_0},\\
&\mathrm{Re} = \frac{\rho b u_0}{\eta_0},\quad \tilde{\boldsymbol{\tau}}_{rr} = \frac{b\bar{\boldsymbol{\tau}}_{rr}}{u_0\eta_0},\quad \tilde{\boldsymbol{\tau}}_{rz} = \frac{d_0\bar{\boldsymbol{\tau}}_{\mathbf{rz}}}{u_0\eta_0},\quad \tilde{\boldsymbol{\tau}}_{zz} = \frac{b\bar{\boldsymbol{\tau}}_{zz}}{u_0\eta_0},\quad \tilde{\boldsymbol{\tau}}_{\theta\theta} = \frac{b\bar{\boldsymbol{\tau}}_{\theta\theta}}{u_0\eta_0},\\
&\alpha = \frac{k_0 u_0}{\eta_0 b},\quad \theta = \frac{\bar{T} - \bar{T}_0}{\bar{T}_0},\quad E_c = \frac{u_0^2}{c_p\bar{T}_0},\quad \Pr = \frac{c_p\eta_0}{k},\\
&S_r = \frac{\rho D K_T(\bar{T}_0)}{\eta_0 T_m(\bar{C}_0)},\quad S_c = \frac{\eta_0}{D\rho},\quad \sigma = \frac{(\bar{C} - \bar{C}_0)}{(\bar{C}_0)},
\end{aligned} \tag{10}$$