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Shape determination of wind-resistant wings attached to an oscillating bridge using adjoint equation method

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ABSTRACT

The purpose of this study is to determine the angle of a wing that is attached to an oscillating bridge located in transient incompressible viscous flows, using the arbitrary Lagrangian–Eulerian (ALE) finite element method and optimal control theory, in which a performance function is expressed by the displacement of the bridge. Currently, some bridges have wings attached to them to prevent oscillation caused by wind flows. When the angle of the wing changes, the state of oscillation also changes. Therefore, the angle of the wing is a very important parameter to consider the minimization of the oscillation of the bridge. In this research, the angle of the wing is determined based on optimal control theory. To minimize the oscillation of a bridge, the performance function is introduced as the minimization index. The performance function is defined by the square sum of the displacements of a bridge. This problem can be transformed into a unconstrained minimization problem by the Lagrange multiplier method. The adjoint equations can be obtained by using the stationary condition of the extended performance function. The gradient used for updating the angle of the wing can be derived by solving the adjoint and state equations. The weighted gradient method is applied as a minimization technique. In this study, the determination of the angle at which the oscillation of the bridge is minimized is presented using this theory. To express the motion of fluids around a bridge, the Navier–Stokes equations described in the ALE form are employed as the state equations. The motion of the bridge is expressed by the motion equations by using the displacements and rotational angle of the body supported by springs. As a numerical study, the optimal control of the angle of the wing is demonstrated at low Reynolds number flows. Thus, the angle of the wing at which the oscillation of the bridge becomes minimum can be determined. Numerical results obtained correspond to the angle of actual bridge wing.

1. Introduction

When a bridge is subjected to wind flows, it is well known that a Karman vortex occurs behind the bridge. The weight and rigidity of recently built bridges have reduced because of the use of high-quality materials; this causes the bridge to unexpectedly oscillate owing to low-velocity wind. The oscillation of the bridge sometimes influences traffic on the bridge. To prevent oscillation caused by wind, some bridges are equipped with wings, as shown in Fig. 1. These are called wind-resistant wings. The optimal angles of these wings are usually determined through wind tunnel experimants.

The purpose of this study is to determine the optimal angle and the shape of wind-resistant wings for keeping the oscillation of the bridge to a minimum, using the adjiont equation methos based on control theory. As shown in Fig. 1, the bridge is a complete two-dimensional structure. The bridge can be modeled as a rigid body having wings that are supported by elastic springs. The oscillation can be evaluated by studying the vertical and rotational movements. The lateral and axial rigidities are extremely stronger than the bending and torsional rigidities. Therefore, only the vertical and rotational oscillations in the vertical plane should be considered for studying the oscillation of the bridge. The wind tunnel experiment is also performed using two-dimensional modeling.

The analysis of a body oscillating because of fluid forces involves moving boundary problem; therefore, the ALE method (e.g., [1–4]) can effectively be applied to resolve this problem. The Navier–Stokes equation is used to express the motion of fluid. The optimal shape determination of an oscillating body equipped with wings is studied here. The angle of the wings is taken as a control variable. The performance function is assumed to be the square sum of the displacements of the body. The vertical oscillation displacement of the body is computed to minimize the performance function under the constraint of the equation of motion. Precise forms of the adjoint equation and the gradient of the performance function with respect to the wing are derived. For discretizing both the equation of motion and the adjoint equation, the finite element method based on the mixed interpolation of the

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Fig. 1. Metropolitan Expressway Route 11 Daiba line.

bubble function and linear function [5,7] is successfully used. For minimizing the performance function, the weighted gradient method is used because the wide range of the initial values can be used in this method.

No paper has been published concerning the determination of the optimal angle of the wing to minimize the oscillatino of bridges. The primary idea employed in this study is presented in Ref. [8]. Plenty of papers dealing with the drag and lift forces for determining the optimal shape of a body located in a viscous fluid flow. Those papers are reviewed in Nakajima and Kawahara [9,10].

In the numerical study, two-dimensional shape determination is discussed, although three-dimensional shape determination has already been published in the papers by Nojima and Kawahara [11,12]; this is because the bridge is constructed as a two-dimensional structure. By using the wind-resistant wings at the optimal angle, we can achieve a considerable reduction in the oscillation displacement. The optimal angle obtained is in good agreement with that of wings actually attached to bridges, which are obtained through the wind tunnel experiments.

2. State equation

2.1. ALE description

Indicial notation and summation convention with repeated indices are used to express the equations in this paper. The bridge is assumed as a rigid body located in fluid. The motion of a body and fluid motion around a body are expressed by the Lagrangian, Eulerian, or reference description. The reference description is independent of the first two descriptions, and it is possible to choose an arbitrary coordinate system. According to these descriptions, the Lagrangian, Eulerian, and reference coordinate systems are expressed by X_i, x_i, and χ_i, respectively. If a function f is an arbitrary physical quantity described in the Euler description, the relation between a real time derivative and reference time derivative is written as follows:

$$\frac{Df}{Dt} = \left.\frac{\partial f(\chi_i, t)}{\partial t}\right|_{\chi_i} + \frac{\partial f(\chi_i, t)}{\partial \chi_j} \cdot \left.\frac{\partial \chi_j(X_i, t)}{\partial t}\right|_{X_i}. \tag{1}$$

If f is a position vector x_i at the present time, Eq. (1) can be written in the following form:

$$\left.\frac{\partial x_i(X_i, t)}{\partial t}\right|_{X_i} = \left.\frac{\partial x_i(\chi_i, t)}{\partial t}\right|_{\chi_i} + \frac{\partial x_i(\chi_i, t)}{\partial \chi_j} \cdot \frac{\partial \chi_j(X_i, t)}{\partial t}|_{X_i} \tag{2}$$

$$= \left.\frac{\partial x_i(\chi_i, t)}{\partial t}\right|_{\chi_i} + w_j \cdot \left.\frac{\partial x_i(\chi_i, t)}{\partial \chi_j}\right|_{X_i}. \tag{3}$$

where

$$w_j = \left.\frac{\partial \chi_j(X_i, t)}{\partial t}\right|_{X_i}. \tag{4}$$

Here, we introduce the relative velocity b_i of the material point for the reference coordinate system:

$$b_i = u_i - \tilde{u}_i = w_j \cdot \left.\frac{\partial x_i(X_i, t)}{\partial \chi_j}\right|_{X_i}, \tag{5}$$

where

$$u_i = \left.\frac{\partial x_i(X_i, t)}{\partial t}\right|_{X_i}, \tag{6}$$

$$\tilde{u}_i = \left.\frac{\partial x_i(\chi_i, t)}{\partial t}\right|_{\chi_i}. \tag{7}$$

Substituting the velocity relation Eq. (5) into the reference time derivative equation, the reference time derivative in the Euler domain is obtained as follows:

$$\frac{Df}{Dt} = \left.\frac{\partial f(\chi_i, t)}{\partial t}\right|_{\chi_i} + b_j \frac{\partial f(\chi_i, t)}{\partial x_j}. \tag{8}$$

Eq. (8) is the time derivative equation of a function f in the ALE method.