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Topological design of electromechanical actuators with robustness toward over- and under-etching

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ABSTRACT

In this paper, we combine the recent findings in robust topology optimization formulations and Helmholtz partial differential equation based density filtering to improve the topological design of electromechanical actuators. For the electromechanical analysis, we adopt a monolithic formulation to model the coupled electrostatic and mechanical equations. For filtering, we extend the Helmholtz-based projection filter with Dirichlet boundary conditions to ensure appropriate design boundary conditions. For the optimization, we use the method of moving asymptotes, where the sensitivity is obtained from the adjoint approach.

Our study shows that the robust filter approach produces topology optimized actuators with minimal length control and crisp structural boundaries. In particular, the minimal length control of both structural features and gap widths avoids common modeling artifacts in topology optimization, i.e. one-element wide structural parts or gaps. It thus leads to physically realizable designs that are robust against manufacturing imprecision such as over- and under-etching.

1. Introduction

Design of multiphysics systems has become increasingly important for a variety of engineering applications. It is challenging to design such systems through engineer's intuition due to complex interactions between physics. Since its early inception [1,2], topology optimization has been applied to a variety of multi-physics systems, particularly MEMS applications [3–10]. This paper presents a robust formulation for topology optimization of nonlinear, coupled electromechanical systems actuated by Coulomb's (electrostatic) forces and is an extension and improvement of the work presented in [10]. The added robust formulation leads to optimized structures with clear black/white (almost no gray) boundaries and with minimal length scale control for both solid and void features. The minimal length scale control improves both mechanical and electrical analysis for topology optimization so that one-node hinges in electrodes or one-element gaps between electrodes that otherwise commonly exist in optimized designs are avoided.

Electrostatics is a simple case of electromagnetism where an electric field is considered as quasistatic due to stationary electric charges [11]. Most of the widely used MEMS devices use the electrostatic phenomenon for actuation, such as comb-drive actuators and sensors consisting of integrated capacitors [12,13]. In a quasistatic electric field, a structure will be subjected to electrostatic force due to induced charges on structural surfaces. This electrostatic force in turn leads to structural deformation. Because the deformation of the structure influences the electric field and the resulting electrostatic force, the coupling between the electric field and the structural displacement must be considered simultaneously [6,8]. In this paper, our analysis is based on a monolithic formulation of the coupled electromechanical analysis [10], rather than typical staggered analyses for coupled problems. The minimal length of both solids and gaps in optimized designs is obtained by solving three sets of such coupled electric and elastic equations with the material density filtered by the Helmholtz partial differential equation based filter. Our study finds that such obtained minimal lengths in optimized designs agree remarkably well with minimal length predicted through the numerical approach [14] or an analytical formula (derived in the Appendix).

The remainder of this paper is organized as follows. Section 2 reviews the monolithic formulation of electromechanical analysis. Section 3 describes the robust formulation of topology optimization under coupled electromechanical governing partial differential equations (PDEs). Section 4 presents how Helmholtz PDE filter under Dirichlet boundary conditions can be implemented. Section 4 details the numerical results on the optimization of an electrostatic displacement inverter and a gripper. The paper is concluded in Section 5.

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2. Monolithic formulation of electro-mechanical equations in the undeformed domain

This paper adopts a monolithic approach for coupled electromechanical analysis first suggested in Ref. [10]. For the sake of self-containedness, we briefly outline this approach in the following. For details, see [10]. This approach is amenable to topology optimization by avoiding many obstacles in the usual staggering analysis approach to coupled problems. More specifically,

- It allows unified equations for modeling both semi-conductors (e.g. silicon) and insulators (e.g. air), thus avoiding alternating physics as in the staggering analysis approach. It uses SIMP material interpolation functions for three material properties in the unified domain: Young's modulus C in the linear elasticity equation, generalized permittivity $\tilde{\varepsilon}$ in the electric Poisson equation, and the permittivity ϵ for the electrostatic force calculations.
- The electrostatic forces are calculated by volume integration instead of usual surface integration of Maxwell's stress tensor due to the absence of explicit representation of structural boundary in topology optimization.
- Governing equations are transformed from the deformed domain to the undeformed domain using the deformation tensor so that no re-meshing or mesh morphing is required in the optimization process.

Using the generalized permittivity $\tilde{\varepsilon}(\boldsymbol{x})$, we can set up the electric equation as,

$$\nabla_{\boldsymbol{x}} \cdot (\tilde{\varepsilon}(\boldsymbol{x})\nabla_{\boldsymbol{x}} p) = 0 \quad \text{in}\, {}^{t}\Omega(\boldsymbol{u}), \tag{1}$$

where ${}^{t}\Omega(\boldsymbol{u})$ represents the deformed domain and p is the electric potential. The generalized permittivity is so chosen that it can model both semi-conductor and insulator simultaneously (cf. [10]). For semi-conductors, a constant potential exists on all surfaces. The particular form of the permittivity interpolation shall be discussed later.

The linear elasticity equation including prestress from the Maxwell's stress tensor is

$$\begin{cases} \nabla_{\boldsymbol{x}} \cdot \boldsymbol{T} + \nabla_{\boldsymbol{x}} \cdot \boldsymbol{T}_E = 0 \quad \text{in } {}^{t}\Omega(\boldsymbol{u}) \\ \boldsymbol{T} = \boldsymbol{C}\boldsymbol{S} \\ \boldsymbol{S} = \frac{1}{2}(\nabla_{\boldsymbol{x}}^T \boldsymbol{u} + \nabla_{\boldsymbol{x}} \boldsymbol{u}) \end{cases}, \tag{2}$$

where $\boldsymbol{T}_E$ is the Maxwell's stress tensor, $\boldsymbol{T}$ is the stress, $\boldsymbol{S}$ is the strain, $\boldsymbol{u}$ is displacement, and the deformation-independent constitutive matrix is denoted as $\boldsymbol{C}$. Note that for the stress, we assume geometrically linear analysis, i.e. we neglect changes of surface areas, volumes and mass densities between deformed and undeformed structural domains [15]. Hence, we have

$$\nabla_{\boldsymbol{x}} \cdot \boldsymbol{T} = \nabla_{\boldsymbol{X}} \cdot \boldsymbol{T}, \tag{3}$$

where $\boldsymbol{x}$ and $\boldsymbol{X}$ represent space coordinates after and before the deformation, respectively. The Maxwell's stress tensor is calculated as follows

$$T_E = \varepsilon(\boldsymbol{x})\left(\boldsymbol{E}\boldsymbol{E} - \frac{\boldsymbol{E}\cdot\boldsymbol{E}}{2}\boldsymbol{I}\right), \tag{4}$$

with the electric field $\boldsymbol{E} = -\nabla_{\boldsymbol{x}} p$.

Combining (1) and (2), we obtain the following weak form of the electric and elastic equations in the deformed domain: find p and $\boldsymbol{u}$ such that

$$\int_{{}^{t}\Omega} (\nabla_{\boldsymbol{x}} \delta p)^T \cdot (\tilde{\varepsilon}(\boldsymbol{x})\nabla_{\boldsymbol{x}} p)\, d\Omega = 0, \tag{5}$$

$$\int_{{}^{0}\Omega} \delta\boldsymbol{S}^T \cdot T\, d\Omega = -\int_{{}^{t}\Omega} \delta\boldsymbol{S}(\boldsymbol{u}, \delta\boldsymbol{u})^T \cdot \boldsymbol{T}_E\, d\Omega, \tag{6}$$

where δp is the test function for the electric potential p, $\delta\boldsymbol{S}$ is the test function (virtual strain) for strain $\boldsymbol{S}$ with $\delta\boldsymbol{S}(\boldsymbol{u}) = \frac{1}{2}(\nabla_{\boldsymbol{X}}\delta\boldsymbol{u}^T + \nabla_{\boldsymbol{X}}\delta\boldsymbol{u})$ and $\delta\boldsymbol{S}(\boldsymbol{u}, \delta\boldsymbol{u}) = \frac{1}{2}(\nabla_{\boldsymbol{x}}\delta\boldsymbol{u}^T + \nabla_{\boldsymbol{x}}\delta\boldsymbol{u})$. Note that the linear structural potential energy is represented directly in the undeformed domain, as assumed earlier. We can transform the other integral forms from the deformed domain into the undeformed domain via the deformation tensor $\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$. Using $\nabla_{\boldsymbol{x}}\boldsymbol{u} = \boldsymbol{F}^{-T}\nabla_{\boldsymbol{X}}\boldsymbol{u}, \nabla_{\boldsymbol{x}} p = \boldsymbol{F}^{-T}\nabla_{\boldsymbol{X}} p$, $\int_{{}^{t}\Omega}()\, d\Omega = \int_{{}^{0}\Omega}()\|\boldsymbol{F}\|\, d\Omega$ and $\delta\boldsymbol{S}(\boldsymbol{u}, \delta\boldsymbol{u}) = \frac{1}{2}\left((\boldsymbol{F}^{-T}\nabla_{\boldsymbol{X}}\delta\boldsymbol{u})^T + \boldsymbol{F}^{-T}\nabla_{\boldsymbol{X}}\delta\boldsymbol{u}\right)$, we thus have the weak form in the undeformed domain, find $p \in P$ and $\boldsymbol{u} \in U$ such that

$$\int_{{}^{0}\Omega} (\nabla_{\boldsymbol{X}}\tilde{p})^T (\boldsymbol{F}^{-1}\tilde{\varepsilon}(\boldsymbol{X})\boldsymbol{F}^{-T})\nabla_{\boldsymbol{X}} p \|\boldsymbol{F}\|\, d\Omega = 0, \quad \forall \tilde{p} \in P_0, \tag{7}$$

$$\int_{{}^{0}\Omega} \tilde{\boldsymbol{S}}^T \cdot \boldsymbol{T}\, d\Omega + \int_{{}^{0}\Omega} \tilde{\boldsymbol{S}}(\boldsymbol{u}, \tilde{\boldsymbol{u}})^T \cdot \boldsymbol{T}_E \|\boldsymbol{F}\|\, d\Omega = 0, \quad \forall \tilde{\boldsymbol{u}} \in U_0, \tag{8}$$

where

$$P = \{p | p \in H^1(\Omega), p = \bar{p} \quad \text{on} \quad \Gamma_p\},$$

$$P_0 = \{\tilde{p} | \tilde{p} \in H^1(\Omega), \tilde{p} = 0 \quad \text{on} \quad \Gamma_p\},$$

$$U = \{\boldsymbol{u} | \boldsymbol{u} \in H^1(\Omega), \boldsymbol{u} = \bar{\boldsymbol{u}} \quad \text{on} \quad \Gamma_u\},$$

$$U_0 = \{\tilde{\boldsymbol{u}} | \tilde{\boldsymbol{u}} \in H^1(\Omega), \tilde{\boldsymbol{u}} = 0 \quad \text{on} \quad \Gamma_u\},$$

The above weak form is solved by the finite element method in this paper. Upon discretization, it leads to residual equations $\boldsymbol{R}(\boldsymbol{u}) = 0$ corresponding to the non-linear finite element implementation of eqs (7) and (8).

3. Robust topology optimization

In the original work on the monolithic topology optimization formulation for electrostatic mechanism design [10], it was quite a challenge to enforce strict length-scales in gap regions. Obviously, one element wide gap regions with significant jumps in electric potential do not represent physical reality well. In order to partially alleviate this problem, Ref. [10] suggested to use the modified Heaviside projection scheme [16], a scheme that for simple compliance problems works very well and ensures minimum length scale control for void regions. For the electromechanical actuator design problem this scheme did ensure finite gap regions to a certain extent, however, problems with enforcing strictly solid-void designs resulted in somewhat unsatisfactory modeling of the electric field in gap regions. Also, the modified Heaviside filtering only controls void length scales, hence it was not able to prevent thin and non-physical hinge regions.

Lately, so-called robust filtering approaches have shown great promise [17,14] in preventing small details and ensuring finite length scales for minimum compliance and compliant mechanism design problems. Apart from ensuring strict control of both solid and void length scales, numerical experiments indicate that the robust filtering concept, that entails optimization of three different design realizations (the blue-print design as well as the under- and over-etched realizations), yields an intrinsic penalization of gray regions.

Based on above observations, we find it worthwhile to revisit the challenging problem of electrostatic compliance mechanism design and to combine it with the newest findings in robust topology optimization approaches to result in a design methodology that ensures physically meaningful simulations and results.