Robust structural topology optimization considering boundary uncertainties
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1. Introduction

The study of topology optimization has received tremendous attention since the pioneering work of Bendsoe and Kikuchi [1]. In general, topology optimization aims to optimize the material layout of a structure within a given design space under prescribed load and boundary conditions such that some design criteria can be satisfied. Nowadays, numerous methods have been proposed for topology optimization and it has been recognized as a very powerful tool to help engineers obtain innovative conceptual designs which cannot be created easily through size and shape optimizations.

Traditionally, topology optimizations are often carried out in a deterministic framework, where it is assumed that the parameters (e.g., material properties, applied loads and structural geometries) involved in the problem can be determined exactly. However, uncertainties of these parameters are unavoidable in real-world applications due to incomplete information, observation errors and manufacturing imperfections, etc. Furthermore, it is also well known that solutions to optimization problems may exhibit remarkable sensitivity to parameter perturbations. This may lead to the performance of the actual structure far from optimal or cannot meet the design requirements. Therefore it is of great importance to take the effect of uncertainty into consideration for optimal topology designs of structures.

Recent years have witnessed a growing interest in carrying out topological designs of structures in the context of robust optimization and reliability-based optimization. Ben-Tal and Nemirovski [2] proposed a semi-definite programming based approach for robust topology optimization of truss structures considering uncertain load conditions. Guest and Igusa [3] considered the topology optimization with uncertainty in the magnitude and location of the applied loads and with small uncertainty in the location of the structural nodes. Kogiso et al. [4] addressed the optimal designs of compliant mechanisms considering the uncertainty of the direction of the driving force. Chen et al. [5] proposed a robust design method for structures with minimum compliance and compliant mechanisms via the level set method. In this work, the uncertainties of applied loads and material properties are considered. de Gournay et al. [6] developed a level set-base approach for the designs of robust structures with minimum compliance accounting for uncertain loads. Level set-based topology optimization has also been employed by Dunning et al. [7] to account for load uncertainties. Cherkaev and Cherkaev [8] pointed out that for some specific forms of load uncertainty, robust optimization of worst-case structural compliance can be reduced to an elasticity problem with mixed nonlinear boundary condition, which may have multiple solutions. When the admissible load can be described by an ellipsoid centered at zero, Takezawa et al. [9] showed that the corresponding topology optimization problem aiming at minimizing the worst-case structural compliance can be formulated equivalently as an eigenvalue optimization problem. Recently, Brittain et al. [10] also considered the robust optimization of continuum structures under a specific form of load uncertainty in a multi-scale framework. Furthermore, for research works related to the reliability-based topology optimization, we refer the readers to [11] and the reference therein for details. It is worth noting that in most of the aforementioned works, only uncertainties of the external load are considered.
Besides load uncertainties, the uncertainty of structural geometries, always characterized by the shape deviation from the designed one, is also ubiquitous in real-world engineering practice. As shown in [12,13], the geometric uncertainties usually result from the manufacturing imperfections, operational wear or degradation of the surface material under erosive environments and may have great impacts on the performances of the structure especially when the considered length scale is small. Therefore, it is of crucial importance to take the effect of geometrical uncertainty into consideration to ensure the robustness and/or the reliability of an optimal topology design. Compared with load uncertainties, however, considering geometry uncertainties is much more difficult and complicated. The challenges come from both the computational and theoretical aspects. As regard to the computational aspect, unlike in the case of load uncertainties, where structural responses are explicit linear functionals of the applied loads (for linear elastic structures), structural responses are always dependent on the shape of the structural boundary in an implicit and highly nonlinear way. Since robust optimization problems are usually formulated in the form of Bi-level program and the lower level optimization problem must be solved many times to find the worst-case scenario of uncertainty and the corresponding structural response, it can be expected that the computational effort involved in the robust optimization with geometry uncertainties will be much larger than that when only load uncertainties are considered. The challenge from the theoretical aspect is associated with the global optimality of the lower program in the Bi-level formulation of the robust optimization problem. It results from the fact that the true worst-case structural response (corresponding to the global optimal solution of the lower level program) cannot be found with full confidence if finding the global optimal solution to the lower level program cannot be guaranteed (see Fig. 1 for reference)! However, since structural responses are in general non-convex functionals of the structural shape, global optimal solutions cannot be expected at least theoretically.

Sigmund [14] and Wang et al. [15] proposed a morphology-based filter method to address the topology optimization problem considering uniform manufacturing errors. “Erode” and “dilate” operators are employed to model the under-etching and over-etching manufacturing errors, respectively. To overcome the limitation of only accounting for uniform geometry imperfections, Schevenels et al. [16] extended the above method to the case where non-uniform manufacturing errors are considered. In this work, the authors followed a probabilistic approach to formulate the optimization problem and used a random Heaviside projection threshold to represent the non-uniform boundary perturbations. Since Monte Carlo simulation is employed to obtain the required mean value and standard deviation of the structural response, the corresponding computational cost is high. Recently, Chen and Chen [17] also developed a level-set based probabilistic approach to shape and topology optimization under geometric uncertainty. In the proposed method, Karhunen-Love expansion is utilized to model the geometry uncertainty quantitatively and Multivariate Gauss quadrature is used to transform the considered problem into a deterministic one, in which the corresponding objective function is a weighted summation of the statistics of the structural response at some sampling points. Numerical examples presented in this paper demonstrated that promising robust optimal solutions can be obtained by the proposed approach.

In the present paper, a topology optimization method considering the uncertainty of boundary variations has been proposed based on the level set framework. Unlike the works of Schevenels et al. [16] and Chen and Chen [17], where the boundary uncertainties were quantified with a probabilistic description, in the present study, the boundary uncertainty is only assumed to be unknown-but-bounded and therefore the statistic information of the boundary variation (sometimes difficult to be obtained) is not required. Accordingly, the corresponding statistical analysis of the structural response, which is very computationally expensive, is also avoided. Also in contrast to the worse-case design problem formulated in Sigmund [14], where only uniform boundary variation was considered, arbitrary boundary variation (with necessary regularity) with given amount of total volume can be accounted for by the proposed problem formulation. The main contribution of the present work is that under some reasonable assumptions, we have formulated the considered problem as a computationally tractable single-level program, which can be solved efficiently with the well-established topology optimization algorithms. This treatment not only reduces the solution effort substantially but also circumvents the difficulty posed by the aforementioned issue of global optimality.

The present paper is organized as follows. Section 2 focuses on the mathematical formulation of the considered problem. In Section 3, shape derivatives required for the numerical solution algorithm are derived. Three numerical examples are presented in Section 4 for demonstrating the effectiveness of the proposed mathematical formulation and the numerical solution approach. Comparisons with the results obtained by deterministic problem formulation are also made in this section. Finally, some concluding remarks are presented in Section 5.

2. Optimization problem formulation

In this section, firstly, the Bi-level formulation for the considered robust structural topology optimization will be presented. Then we will show how to transform this Bi-level formulation into a computationally tractable single-level formulation.

2.1. Bi-level formulation for robust topology optimization considering the uncertainty of boundary variations

The robust structural topology optimization problem considered in the present study is to find the optimal topology of a structure in a prescribed design domain under material volume constraint such that the worst-case structural compliance under given amount of boundary variation is minimized. Within the level set framework, the optimization problem can be formulated as follows:

\[ \max_{x \in [a, b]} g(x) \leq c \]

**Fig. 1.** Local optimum and global optimum in robust optimization. \( g(G) \) represents the worst-case structural response. The uncertain parameter \( x \) varies in the interval \([a, b]\). \( c \) is a upper bound imposed on the worst-case structural response.