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A strong discontinuity approach on multiple levels to model solids at failure

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ABSTRACT

This paper presents a modification of the well established strong discontinuity approach to model failure phenomena in solids by extending it to multiple levels. This is achieved by the resolution of the overall problem to be solved into a main boundary value problem and identified sub-domains based on the concepts of domain decomposition. The initiation of those sub-domains is based on the detection of failure onset within finite elements of the main boundary value problem which takes place at the process zone in front of the propagating cracks. Those sub-domains are subsequently adaptively discretized during run-time and comprise the so called sub-boundary value problem to be solved simultaneously with the main boundary value problem. To model failure, only the sub-elements of those sub-boundary value problems are treated by the strong discontinuity approach which, depending on their state of stress, may develop strong discontinuities to be understood as jumps in the displacement field to model cracks and shear bands. Due to its resolution into many sub-elements, the single finite element of the main boundary value problem can therefore simulate a single propagating strong discontinuity arising in quasi-static problems as well as the propagation of multiple propagating strong discontinuities arising for simulations of crack branching in brittle materials undergoing dynamic failure. Whereas the advantages of the strong discontinuity approach in the form of its efficiency by statically condensing out the degrees of freedom related to the failure zone as well as its applicability to use standard displacement based, mixed, and enhanced formulations for the underlying finite element are kept, new challenges arise due to its proposed modification. Firstly, the solutions of the different sub-boundary value problems must be transferred to the main boundary value problem, which is achieved in this work based on concepts of domain decomposition. Secondly, since multiple strong discontinuities might propagate over the boundaries of the sub-boundary value problem, the applied boundary conditions must take into account the appearance of possible jumps in the displacement fields arising from the solution of the sub-boundary value problem itself. It is shown that for single propagating cracks arising in problems of quasi-static failure only minor differences are obtained through the proposed modification. For the simulation of solids undergoing dynamic fracture the modification allows though to predict the onset of crack branching without the need for any artificial crack branching criterion. A close agreement with experiments of the simulation results in terms of micro- and macro branching in addition to studying certain key parameters like critical velocity, dynamic stress intensity factor, and the strain energy release rate at branching is found.

1. Introduction

The modeling of solids at failure remains one of the most challenging topics in computational mechanics. In addition to a raised understanding of traditional engineering materials it allows to contribute to the search for new advanced materials for which the determination of failure is of highest importance. The incorporation of cracks or shear bands as the characteristic microstructure of failing solids, commonly referred to as strong discontinuities representing jumps in the primary unknowns such as the displacement field for purely mechanical problems, into numerical frameworks such as the finite element method are possible only through highly advanced frameworks. Still, all numerical methods have in common the challenge of predicting the onset of failure and the determination of the direction of the propagating strong discontinuity if resolved discretely. This complexity is further raised when dynamic instabilities are accounted for, which arise in problems of dynamic fracture such as for crack branching phenomena to be considered in this work. Early experimental literature about dynamic fracture can be found in experiments by Kobayashi et al. [36], Kobayashi and Ramulu [35], Ramulu and Kobayashi [65] and Ramulu et al. [66] on thin sheets of a brittle material Homalite-100 who attributed the phenomena of crack branching to the

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critical stress intensity factor at the crack-tip. Ravi-Chandar and Knauss [69,67,68] with their series of experiments of dynamic fracture also found similar relations of crack branching with the critical stress intensity factor. More recent experiments by Fineberg et al. [22,23], Sharon et al. [72] as well as Sharon and Fineberg [71] performed on the brittle material PMMA shed more light on the instabilities associated with the fast moving crack in dynamic fracture where micro- branching phenomena were observed before the main branching, which tend to take place at a critical main crack-tip velocity as suggested by Yoffe [79].

A large number of numerical techniques have been developed over the past decades, all with certain advantages for different applications. *Adaptive remeshing techniques* such as those in Ortiz and Quigley [60], Pandolfi and Ortiz [61], Heintz et al. [31], or Miehe and Gürses [50] in n_{dim} dimensions for example align their finite element boundaries along the propagating crack, which on the one hand are computationally expensive but on the other hand do allow for the proper accuracy at highly refined crack tips by increasing the number of critical nodes at the failure zone based on some error estimator. The failure for such approach is modeled by the insertion of $(n_{\text{dim}} - 1)$ dimensional finite elements in between the elements of which the specimen under consideration consists, along which specific traction-separation laws are used. The first approach, which allowed for the propagation of strong discontinuities through the individual finite element goes back to the seminal work of Simo et al. [73] and is commonly referred to as the *strong discontinuity approach*. This framework, later extended to the two and three dimensional continuum within the infinitesimal and the finite deformation setting in Armero and Garikipati [5], Oliver [58], Mosler and Meschke [55], or Oliver et al. [59] relies on a multiscale approach as outlined in Armero [1,2] through which the overall boundary value problem is divided into a global problem, as the standard problem to be solved, and a local problem through which each point in the global problem is capable of developing a strong discontinuity. Extensions to multiphysics problems are performed in Steinmann [75] or Callari and Armero[15] for porous media and in Linder et al. [44], Linder and Miehe [43], Linder [40] for piezo- and ferroelectric ceramics where in addition to the jumps in the displacement field also jumps in the electric potential arise. The methodology allows for the development of new finite elements by the incorporation of certain separation modes directly into the finite element framework. This is achieved in Ehrlich and Armero [20], Amero and Ehrlich [4] for beams and plates, in Linder and Amero [41] and Armero and Linder [6] for the 2D continuum, in Armero and Linder [7] and Linder and Armero [42] for problems in dynamic fracture, in Linder et al. [44] for electromechanical coupled problems, and in Armero [3] for anti-plane/torsional problems. Alternative approaches to the strong discontinuity concept are the *extended finite element method* [12,54,76,77,29,48], which also allows for the propagation of the strong discontinuities through the individual finite elements but with a higher computational cost or *phase field models* [24,14,52] in which the strong discontinuity is smeared over a finite width rather than captured in a discrete way, which allows for the simulation of complex crack patterns but requires a highly dense finite element discretization.

Focusing on a modification of the way how the strong discontinuity approach is applied to model failure in solids in this work, its advantage over alternative strategies given by its computational efficiency, which comes from relatively coarse finite element meshes which can be used to describe the failure process, shall be kept. Still, a certain density of the meshes is needed in particular in areas where materials failure is probable, like around notches or at corners. The goal of this work is to retain a coarse finite element mesh regardless of possible areas of fracture and instead introduce a second level of computation at a finite element where fracture takes place with their coupling achieved through the method of domain decomposition found in Park and Felippa [62], Park et al. [63], or Lloberas-Valls et al. [46], which allows the partitioning of a domain into certain sub-domains where those are connected to the main domain by a coupling based on the Lagrange multiplier method. Applications for heterogeneous materials can be found in Markovic et al. [47], Niekamp et al. [57], or Lloberas-Valls et al. [45]. The different length scales considered in this work are finitely separated, which makes this class of method different from the classical homogenization methods in [26,27,51,49,39,21] where the length scales considered are substantially separated with a coupling introduced at each quadrature point where micro-scale computations are performed on a representative volume element. An attempt to model failure with a discrete resolution of the strong discontinuities with these classical homogenization methods can be found in Belytschko et al. [13] and Song and Belytschko [74] where the power theorem [32] is extended to account for failure within the representative volume element for coupling between the different scales.

In this work, a new sub-domain is introduced at each finite element of the main boundary value problem in front of the crack tip representing the process zone of the propagating crack. Such, so called sub-boundary value problem is treated by the strong discontinuity approach and allows for the modeling and prediction of failure. In particular, single or multiple strong discontinuities can start to grow within the sub-boundary value problem in that way allowing for crack tips as well as the presence of multiple cracks within the interior of the finite elements of the main boundary value problem to e.g. model the bi- and trifurcation phenomena related to crack branching. Challenges arise when the strong discontinuities propagate out over the boundaries of the introduced sub-boundary value problem into the next sub-boundary value problem being initiated automatically during run-time. In such a scenario the accurate application of the chosen boundary conditions at the sub-boundary value problem is of major importance. Displacement based boundary conditions, e.g. linearly distributed over the boundary of the sub-boundary value problem based on the nodal values arising from the main boundary value problem, become invalid due to the jump in the displacement field also present along the sub-boundary value problem's boundary [17]. Therefore, an emphasis of the present work lies in the development of proper boundary conditions at the sub-boundary value problem level. It is furthermore shown that whether the crack continues as a single discontinuity or divides itself into two or three crack branches, is solely determined by the stress state at the sub-elements so that no crack-tip velocity criterion is needed to determine the branching scenario.

The outline of the rest of this paper is as follows. Section 2 presents the changes introduced to the strong discontinuity approach when extending it to multiple levels. After its derivation for the quasi-static and the fully transient setting within the continuum framework in Section 2.1, it is extended to the discrete finite element framework in Section 2.2. Thereafter, the coupling between the multiple levels arising in the form of the main boundary value problem and the sub-boundary value problems is achieved in Section 2.3 by matching the virtual energy on both levels. An emphasize is directed in Section 2.4 towards the determination of modified boundary constraints accounting for the presence of displacement jumps along the boundary of the sub-boundary value problem, which becomes particularly challenging in the presence of multiple strong discontinuities propagating out of the sub-boundary value problem. Section 3 outlines in detail the numerical implementation of the boundary value problems on both levels and provides a detailed investigation of the numerical coupling procedure. Representative numerical simulations of problems within the quasi-static case are shown in Section 4. After assuring