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A stabilized Fractional Step, Runge–Kutta Taylor SPH algorithm for coupled problems in geomechanics

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ABSTRACT

Failure of geomaterials with pores filled with fluids is an important research area in both civil and geological engineering. Many finite element formulations for coupled problems present difficulties such as overestimating failure loads, or mesh alignment dependence resulting on spurious failure mechanisms. Moreover, the spaces where field variables are approximated have to fulfill additional requirements ensuring stability. Stress-velocity-pore pressure formulations in FE analysis provide accurate results for wave propagation and failure analysis. However, finite elements present important limitations when deformations are large. The purpose of this paper is to present a stabilized Fractional Step, SPH algorithm which combines the advantages of the SPH method for large deformation problems with those of the Taylor Galerkin algorithm used within the finite element framework.

1. Introduction

There are special cases in solid mechanics where the material has interconnected voids which are filled with one or more fluids. An example of great interest in civil and geological engineering is that of soils and rocks, where the overall mechanical behavior is governed by this coupling.

Engineers have to predict the behavior of soil and rock geostructures such as earth dams, dykes, foundations and natural slopes both under design loads and at failure. The accurate determination of failure loads and mechanisms will help to reinforce the structure if necessary. Frequently, failure is a dynamic process, and even in cases where triggering of failure is quasi static, the analysis has to take into accounts accelerations occurring in the post failure regime.

The coupling which exists between pore fluids and the solid skeleton results on a mathematical model where displacements (or velocities), effective stresses and pore pressures are necessary to fully characterize the problem.

Coupled formulations, when discretized, may result on instabilities caused by the nature of functional spaces where the governing variables are approximated. In order to avoid them, Babuska–Brezzi conditions [1,2] have to be satisfied.

In finite element analysis, it is well known how failure mechanisms and limit loads may depend on the type of element used. Indeed, most of displacement based formulations, specially those using low order elements such as triangles and tetrahedra present problems of (i) overestimating failure loads or locking, (ii) providing spurious failure mechanisms (mesh alignment effects), and (iii) not being able to provide accurate results in cases where shocks are propagating because the numerical diffusion and dispersion properties of the schemes.

The solutions are: (i) using mixed elements including displacements and pressures as main variables which avoids locking but not alignment effects [3], (ii) use enhanced strain methods [4], which provide an excellent solution for both locking and alignment problems. However, if enhanced strain elements are used for coupled problems in geomechanics, special stabilization techniques have to be used [5,6].

As an alternative, mixed stress-velocity formulations provide good accuracy for computation of limit loads and failure mechanisms, together with good propagation properties. It is worth mentioning here the pioneering works of Cantin et al. [7] and Zienkiewicz and Boroomand [8], and those more recent of Cervera et al. [9,10] and Codina [11].

The authors have proposed – within the FE framework – a Taylor Galerkin algorithm formulated in stress, velocity and pore pressures for non linear solid and soil dynamics problems [12–15]. It consists of casting the balance of momentum and constitutive equations as a system of first order hyperbolic equations. The Taylor Galerkin method was introduced independently by Donea [16] and Lohner et al. [17], and applied to fluid dynamics problems by Peraire et al. [18], and Donea et al. [19]. The interested reader can find a detailed description in the text by Zienkiewicz and Taylor [20].

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Concerning failure, it involves large deformations. Here, meshless Lagrangian methods such as the Smooth Particle Hydrodynamics (SPH), can deal with the problem in a natural manner, without the difficulties encountered in mesh based methods where mesh refinement has to be performed.

The Smoothed Particle Hydrodynamics (SPH) is the first meshless method which has been proposed. It was applied to model astrophysical problems [21,22]. From there, it was extended to classical hydrodynamics problems [23]. Today SPH is used in many areas, among which it is worth mentioning magneto-hydrodynamics [24], multi-phase flows [25], viscous flows [26], quasi-incompressible flows [27,28], flows through porous media [29], metal-forming [30], impact problems [31], elastodynamics problems [32], fast landslide propagation [33,34] and fluid structure interactions [35]. Recently and for the first time, SPH has been applied to soils problems involving soil–water interaction [36] and failure [37]. It is also worth-mentioning the work of Vidal et al. who proposed a stabilized updated lagrangian SPH [38].

Concerning the disadvantages and difficulties presented by the SPH method we can mention (i) The boundary deficiency problems which can be solved by applying a normalization to the Smoothed Hydrodynamics method [39], and (ii) the tensile instability which appear in dynamics problems with material strength [40–42].

Concerning the coupling between the solid skeleton and the pore fluid, we will show in the paper that if the compressibility of solid grains and water are very small, and the permeability tends to zero, the structure of the model is similar to that of incompressible solids and fluids, including a zero divergence condition for the velocity field. We will use here a Fractional Step technique proposed by Chorin [43] which allows the use of the same approximation spaces for velocities and pore pressures.

The purpose of this paper is to present a Fractional Step algorithm which can be used to analyze the behavior of saturated geostructures. It is an extension of previous work done by the authors within the FE framework.

The main goals are the following:

(i) Development of a Fractional Step (FS) algorithm for coupled problems in geomechanics avoiding instabilities. The spatial discretization technique used is the SPH.
(ii) Using a Taylor-SPH model in the first step of the FS algorithm, which avoids tensile instability problems.

The paper is structured as follows: After a first introductory section, we present in Section 2 the mathematical model describing the coupled behavior of geomaterials in terms of effective stresses, velocities and pore water pressures. Section 3 is devoted to describe the proposed Fractional Step, Taylor SPH algorithm. Finally, we present in Section 4 several examples showing the behavior of the proposed model, including a case for which there exists an analytical solution.

2. Mathematical model

Soils and rocks are geomaterials with voids which can be filled with water, air, and other fluids. They are, therefore, multiphase materials, exhibiting a mechanical behavior governed by the coupling between all the phases. Pore pressures of fluids filling the voids play a paramount role in the behavior of a soil structure, and indeed, their variations can induce failure. If we consider the soil as a mixture, we will have equations describing: (i) balance of mass for all phases, i.e., solid skeleton, water and air, in the case of non saturated soils (ii) balance of linear momentum for pore fluids and for the mixture, and (iii) constitutive equations. A crucial point is the choice between eulerian and lagrangian formulations. In soil mechanics, the approach followed most often is mixed, lagrangian for the skeleton and eulerian for the relative movement of the pore fluids relative to the soil skeleton. In many occasions, convective terms can be neglected. We will cast the model in terms of effective stresses, which are responsible for the soil skeleton deformation. Taking tractions as positive, we will have:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - p_w \mathbf{I} \tag{1}$$

where $\boldsymbol{\sigma}'$ is the effective stress tensor, p_w the pore pressure, and $\mathbf{I}$ the second order identity tensor.

The first mathematical model describing the coupling between solid and fluid phases was proposed by Biot [44,45] for linear elastic materials. This work was followed by further development at Swansea University, where Zienkiewicz and coworkers [46–49] extended the theory to non-linear materials and large deformation problems. It is also worth mentioning the work of Lewis and Schrefler [50], Coussy [51] and de Boer [52].

We will describe here a variation of the $u - p_w$ model of Zienkiewicz and coworkers [47] which has been written in terms of velocities. We will refer to it as $v - p_w$. The model will be formulated using an updated Lagrangian framework. The basic idea is that at each time station we use the actual configuration as a reference of a Lagrangian analysis. After all relevant increments of this time step the coordinates of the nodes are updated, providing a new configuration which will be used as initial in the following time step.

Concerning objective rates of the involved variables, we will use the Jaumann-Zaremba rate of the Cauchy stress tensor $\boldsymbol{\sigma}$, and the rate of deformation tensor $\mathbf{d}$, given by:

$$\overset{\nabla}{\boldsymbol{\sigma}} = \frac{D\boldsymbol{\sigma}}{Dt} + \boldsymbol{\sigma}\cdot\boldsymbol{\omega} - \boldsymbol{\omega}\cdot\boldsymbol{\sigma} \tag{2}$$

And

$$\mathbf{d} = \mathrm{grad}_{sym}\,\boldsymbol{v} \quad \text{or} \quad d_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3}$$

where we have introduced the anti symmetric tensor $\boldsymbol{\omega}$, which is usually referred to as the spin tensor, with components given by

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right) \tag{4}$$

If we assume that relative velocities and accelerations of fluids relative to solid skeleton are small, the model can be cast in terms of velocity of the solid skeleton $\boldsymbol{v}$ and Darcy's velocities of pore fluid $\boldsymbol{w}$. We will consider first the balance of mass and linear momentum equations for the pore water, which will be written as:

$$\mathrm{div}(\boldsymbol{w}) + \mathrm{div}\,\boldsymbol{v} + \frac{1}{Q}\frac{dp_w}{dt} = 0 \tag{5}$$

where we have introduced the mixed volumetric stiffness Q as:

$$\frac{1}{Q} = \left\{\frac{1-n}{K_s} + \frac{n}{K_w}\right\} \tag{6}$$

Where n is the porosity and K_s and K_w are the volumetric stiffnesses of the soil particles and water respectively.

The balance of linear momentum for the pore fluid is written as:

$$\rho_w \frac{d\boldsymbol{v}}{dt} + \rho_w\left\{\frac{d}{dt}\left(\frac{\boldsymbol{w}}{n}\right) + \frac{\boldsymbol{w}}{n}\mathrm{grad}\,\boldsymbol{v} + \frac{\boldsymbol{w}_w}{n}\mathrm{grad}\left(\frac{\boldsymbol{w}_w}{n}\right)\right\}$$
$$= -\mathrm{grad}p_w + \rho_w \boldsymbol{b} - \mathbf{k}_{\mathbf{w}}^{-1}\boldsymbol{w} \tag{7}$$

where ρ_w is the density of the pore water, $\boldsymbol{w}$ the Darcy velocity, and $\mathbf{k_w}^{-1}$ the inverse of the permeability tensor. We will assume that permeability is isotropic and can be represented by the scalar k_w.