Journal of the Taiwan Institute of Chemical Engineers 41 (2010) 29–34

Contents lists available at ScienceDirect

Journal of the Taiwan Institute of Chemical Engineers

journal homepage: www.elsevier.com/locate/jtice

Heat transfer analysis of three-dimensional flow in a channel of lower stretching wall

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ARTICLE INFO

Article history:
Received 23 February 2009
Received in revised form 27 March 2009
Accepted 30 March 2009

Keywords:
Channel flow
Viscous fluid
Stretching sheet
Heat transfer
Analytic solution

ABSTRACT

This communication aims to report the heat transfer analysis in generalized three-dimensional channel flow of a viscous fluid over a stretching sheet heated at constant temperature. Energy losses due to viscous dissipation have been taken into account. Highly accurate and purely analytic solution is obtained by homotopy analysis method (HAM). Convergence of the HAM solution is shown through the first table and the residual errors are plotted in the second figure of this article. Effect of the Prandtl number and Eckert number on temperature profiles and heat transfer rate has been discussed in detail.

1. Introduction

In last few decades viscous flow over stretching sheet has received a great interest from chemical engineers due to their technological applications in polymer industry. Such flows have been observed to be of great importance in many processes such as the extrusion of a polymer in a melt-spinning process, the extrude from the die is some time stretched into a sheet to achieve the desired thickness and solidified simultaneously by gradual cooling by direct contact with air or liquid, giving rise to a fluid dynamic problem. In such processes the final product of desired characteristic is obtained by controlling the stretching and cooling processes. The study of flow and heat transfer phenomenon attracts the mathematicians and computer scientists to deal with the governing nonlinear Navier–Stokes equations in order to explore the hidden physics.

In two-dimensional viscous flow over a stretching surface Crane (1970) happened to be lucky enough to find a closed form solution to the governing nonlinear Navier–Stokes equations. However, such fortunes no more persist if one is dealing with the generalized three-dimensional flow over stretching sheet. Wang (1984) studied three-dimensional viscous flow over a flat sheet stretching in two lateral directions and obtained numerical solution. Mehmood and Ali (2006) investigated the three-dimensional flow with heat transfer phenomenon over a sheet stretching in two lateral directions and obtained highly accurate analytic solution for the governing nonlinear equations. After Crane (1970) the problem enjoyed a great attention from researchers in the field. Several extensions have been made by introducing many interesting features such as suction/blowing; MHD, viscoelasticity of the liquid, *etc.* (Ali and Mehmood, 2008; Andersson, 1992; Banks, 1983; Banks and Zaturska, 1986; Chaudhary *et al.*, 1995; Elbashbeshy, 1998; Grubka and Bobba, 1985; Liao, 2003a; Troy *et al.*, 1987; Xu and Liao, 2005; Zakaria, 2004).

In industrial processes such as manufacturing of materials by extrusion processes and heated materials travelling between a feed roll and wind up roll or on a conveyer belt there is strong involvement of heat transfer phenomenon. In manufacturing the polymer sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. In order to achieve the desired characteristics in the final product the process of stretching and cooling must be controlled. This gives rise to the study of heat transfer phenomenon in stretching driven flows over flat heated surface. Keeping this fact in mind Dutta *et al.* (1985) investigated the heat transfer analysis in a flow over a stretching plate subjected to uniform heat flux, Gupta and Gupta (1977) and Chen and Char (1988) studied heat transfer over a flat plate subjected to suction/injection, Dandapat and Gupta (1989) investigated the heat transfer flow in a viscoelastic fluid over a stretching sheet. Recently, Mehmood *et al.* (2008c) studied heat transfer analysis in three-dimensional unsteady viscous flow over a stretching sheet.

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doi:10.1016/j.jtice.2009.03.011

Nomenclature

a,b	velocity gradients
Ec	Eckert number
Ec_x, Ec_y	local Eckert numbers
f, g	dimensionless stream functions
h	width of the channel
Pr	Prandtl number
Re	Reynolds number
T	dimensionless temperature function
u, v, w	velocity components
V_0	suction/injection velocity
x, y, z	space coordinates

Greek symbols

β	$=\frac{b}{a}$, ratio of velocity gradient
η	dimensionless space variable
θ	dimensionless temperature function
λ	dimensionless suction/injection velocity

In most of the above studies the authors considered flow over a flat surface with unbounded domain. Very little attention has been given to the channel flows where the lower plate is a stretching sheet. However, there are few studies present in literature in which the authors have considered channel flows over a stretching sheet. Borkakoti and Bharali (1983) investigated two-dimensional channel flow, Vajravelu and Kumar (2004) studied channel flow in a rotating fluid. Recently, Mehmood and Ali (2008a) studied heat transfer analysis of a rotating viscous flow in a channel. In our present study we aim to investigate the heat transfer phenomenon in a channel flow of a viscous fluid over a flat surface stretching in two lateral directions with different rates as considered by Mehmood and Ali (2008a). The problem is solved analytically by homotopy analysis method (HAM) and the obtained results are highly accurate. The convergence of the solution series has been shown in tabular form.

Homotopy analysis method has newly been proposed by Liao (2003b). Liao (2003b) himself applied the homotopy analysis method to many nonlinear problems and obtained highly accurate results (Liao, 1999, 2003a, 2005, 2006, 2009; Liao and Campo, 2002; Liao and Cheung, 2003; Yang and Liao, 2006). The method (HAM) is being used by many researchers and scientists to deal with nonlinear problems in science and engineering (Abbasbandy, 2008; Abbasbandy and Zakaria, 2008; Allan and Syam, 2005; Bataineh *et al.*, 2008; Chen and Liu, 2009; Domairry and Bararnia, 2008; Ghotbi et al., 2009; Majid *et al.*, 2009; Mehmood and Ali, 2007a,b, 2008a,b; Tan and Abbasbandy, 2008; Wang, 2003; Xu, 2004; Yao, 2009; Ziabakhsh and Domairry, 2009) which prove the validity and usefulness of the method.

2. Flow analysis

2.1. Mathematical description

Consider two infinite parallel flat plates situated at $z = 0$ and $z = h$. The lower plate is a highly elastic membrane and upper is a uniformly porous plate subjected to constant injection. The space $0 < z < h$ is occupied by a viscous fluid. Further the lower plate is maintained at a constant temperature θ_w whereas the fluid has temperature θ_0 such that $\theta_w > \theta_0$ so that heat can flow from plate to fluid. The flow is generated due to the uniform stretching of the lower plate in two lateral directions with different rates. Therefore, following Mehmood and Ali (2007a) the equations governing a three-dimensional flow of a viscous fluid in dimensionless form are given by:

$$f'''' - Re(f'f'' - ff''' - gf''' - g'f'') = 0 \tag{1}$$

$$g'''' - Re(g'g'' - fg''' - f'g'' - gg''') = 0 \tag{2}$$

Subject to the boundary conditions

$$f'(0) = 1, \quad g' = \beta, \quad f(0) + g(0) = 0, \quad f'(1) = 0,$$
$$g'(1) = 0, \quad f(1) + g(1) = \lambda \tag{3}$$

where $'$ denotes the differentiation with respect to η and the dimensionless quantities are defined through

$$\eta = \frac{1}{h}z, \quad u = axf'(\eta), \quad v = ayg'(\eta), \quad w = -ah(f+g), \quad Re = \frac{ah^2}{\nu},$$
$$\lambda = \frac{V_0}{ah}, \quad \beta = \frac{b}{a}, \quad (a \neq 0) \tag{4}$$

in which Re is the Reynolds number, β is the ratio of the velocity gradients, and λ is the dimensionless injection parameter.

Since the flow is three-dimensional therefore it is reasonable to assume the temperature profile of the form $\theta = \theta(x, y, z)$. Therefore the energy equation for such type of flow with viscous dissipation in dimensionless form is given by

$$T'' = Pr[Re(f+g)T' + 4EcRe(f'^2 + g'^2 + f'g' + Ec_x f''^2) + Ec_y g''^2] = 0 \tag{5}$$

Subject to the boundary conditions

$$T(0) = 1, \quad T(1) = 0 \tag{6}$$

where $Pr = \mu Cp/k$, is the Prandtl number, $Ec = \mu a/\rho Cp(\theta_w - \theta_0)$ is the Eckert number, $Ec_x = a^2x^2/Cp(\theta_w - \theta_0)$ and $Ec_y = a^2y^2/Cp(\theta_w - \theta_0)$ are the local Eckert numbers based on the lateral directions x and y, respectively, and the dimensionless temperature is given by

$$T(\eta) = \frac{\theta - \theta_0}{\theta_w - \theta_0} \tag{7}$$

where θ_w and θ_0 are the temperatures of the wall and of the fluid, respectively.

2.2. HAM solution

We use homotopy analysis method to solve the system (1), (2) and (5) subject to the boundary conditions (3) and (6), respectively. To start with HAM one needs to make the initial guess approximations satisfying the boundary data and to choose suitable linear operators (Liao, 2003a,b). We choose the initial guess approximations for velocity and temperature satisfying the boundary conditions (3) and (6) as follows:

$$f_0(\eta) = \eta + \left(\frac{3}{2}\lambda - 2\right)\eta^2 + (1-\lambda)\eta^3 \tag{8}$$

$$g_0(\eta) = \beta\eta + \left(\frac{3}{2}\lambda - 2\beta\right)\eta^2 + (\beta-\lambda)\eta^3 \tag{9}$$

$$T_0(\eta) = 1 - \eta \tag{10}$$

We choose the linear operator for velocity and temperature functions respectively of the form:

$$\mathcal{L}_v = \frac{d^4}{d\eta^4} \tag{11}$$