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Operator- and template-based modeling of solid geometry for Isogeometric Analysis with application to Vertical Axis Wind Turbine simulation

P. Stein [a], M.-C. Hsu [b], Y. Bazilevs [b,*], K. Beucke [c]

[a] Geschwister-Scholl-Strasse 8, 99423 Weimar, Germany
[b] Department of Structural Engineering, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093, USA
[c] President (Rector), Bauhaus-Universität Weimar, Coudraystrasse 7, 99423 Weimar, Germany

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ABSTRACT

This article describes a novel approach to modeling and storage of NURBS-based solid objects for Isogeometric Analysis. The proposed method is based on a procedural description of the modeling process. Creation of geometric objects *as well as the steps of the modeling process* are formulated as a list of simple commands. This provides an abstraction from the often times tedious manual specification of control point locations to create a given geometric object. This operator-based approach, in conjunction with the existing template-based geometry modeling methods, allows one to create complex and multi-level adaptive models. To illustrate our method, we construct the geometry of a Vertical Axis Wind Turbine (VAWT) that is suitable for isogeometric fluid and fluid–structure interaction analysis. A new template is proposed for modeling VAWTs together with a novel algorithm for constructing wind turbine airfoil profile B-Spline curves from point data. The resultant model has a compact representation that makes use of a small number of parameters. A preliminary aerodynamics simulation of a newly constructed VAWT model in 3D under realistic wind conditions and rotation speed is presented.

1. Introduction and previous work

Non-uniform rational B-Splines (NURBS) are a common tool in Computer-Aided Design (CAD), Computer Graphics (CG) and related fields. They are typically used to describe planar and spatial curves and surfaces. The underlying representation of curves and surfaces makes use of control points in conjunction with smooth spline basis functions (B-Splines or NURBS) defined in a parametric domain. In the case of curves, the functions depend on a single parameter and are called univariate splines. In the case of surfaces, the functions depend on two parameters and are called bi-variate splines. The order and the degree of continuity of the basis functions employed determine the properties of the resulting geometric object.

A less common object in the CAD and CG communities is the tri-variate solid. In this case, the underlying spline basis functions depend on three parametric coordinates and one is able to represent three-dimensional shapes with parameterized interior (see [34] for an overview). Such tri-variate representations have gained importance only in recent years. One of the reasons is its usefulness in representing heterogeneous data for medical, science, and engineering applications [34]. Another reason is the introduction of Isogeometric Analysis (IGA) [26], a computational technology that is an alternative to the finite element method (FEM), where bi- and tri-variate representations of CAD geometry are employed directly in the computational analysis of mathematical models described by partial differential equations. The relative merits of IGA, its veracity and its potential are described in a recent book [18] on the subject.

NURBS are a mature technology: a variety of algorithms exist for creating and modifying NURBS curves and surfaces. Numerous software packages provide the means for interactively describing these free-form objects. This, however, is not the case for NURBS solids. Such models have to be defined by the user as a list of model *primitives*, i.e., control points and knot vectors, which define the underlying NURBS basis functions. Volumetric NURBS models require specification of more data than curves or surfaces. As a result, while the manual definition of tri-variate solid geometries is manageable for small models, it is a tedious and error-prone process for models of larger size. Furthermore, the lack of appropriate modeling tools poses a serious problem when model adaptions arise. The list of model primitives does not describe the structural logic of the geometric model, that is, the relations among the model components cannot be represented by a list of primitives [23,42]. Hence, there is no indication which low-level entities have to be modified in order to obtain a desired high-level shape change.

* Corresponding author.
E-mail address: yuri@ucsd.edu (Y. Bazilevs).

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In the recent years a number of modeling approaches for NURBS solids that are suitable for computational analysis were described in the literature. Most of these aim at extending existing CAD functionality and methods. We briefly summarize them here:

- *Surface expansion*: There exist several algorithms that create NURBS surfaces from the combination of single or multiple NURBS curves. Operations such as sweeping, extrusion or ruling are a basic tool in computer-aided geometric design (see e.g., [38,39]). For surface modification, operators such as "taper", "twist", "flatten", and "lift" have been applied to regions of NURBS surfaces. These were specified by location, rather than by requiring the user to indicate subscripts (see [16] for details). These operations may be extended to allow the creation of NURBS solids from NURBS surfaces. Some early work was done in [37] to deform volumetric spline models without having to move each control point manually. However, the implementation of such modeling operations may be challenging. For example, [1] describes the issues involved in sweeping a cross-section along a guidance curve, while [34] deals with the challenges of assigning heterogeneous volumetric attributes to NURBS solids, and the transfer of these attributes under surface expansion operations.
- *Constructive modeling*: One of the fundamental representation schemes for solid objects is Constructive Solid Geometry (CSG). Within this scheme a complex body is composed from basic shape primitives such as spheres, cylinders, cubes, etc. The shapes are combined using regularized Boolean operations, which leads to the final geometry [41]. The biggest challenge for NURBS in this approach is to ensure the correct alignment of the different regular meshes of the shape primitives. In [36], the authors developed a Constructive Solid Analysis framework for CSG using the ideas of the partition of unity [35] and meshless FEM [12].
- *Extraction of parametrizations*: A common problem in geometric modeling is to find a parametrization for a given set of discrete data. Different algorithms that allow to convert discrete data sets into NURBS curves and surfaces are described in [38]. These algorithms employ some form of interpolation or approximation. In [33] the authors describe an approach that allows them to create a trivariate parametrization from a given set of triangular surface meshes. The surface meshes describe the exterior boundary of a solid object and give additional information about the interior layers.
- *Template-based modeling*: Man-made objects, in particular those with a technical origin, possess a certain regularity. They may often be characterized by a handful of parameters. These characteristics may be captured in the form of *templates*. Templates may be defined as a pattern of control points and interpolation functions that describe certain basic shapes, for instance circular arcs. They may be combined in order to obtain more complex shapes. In [49] a template-based approach was developed for IGA of cardiovascular blood flow and fluid–structure interaction (FSI). Templates of blood vessel branching configurations enabled the authors to construct models of patient-specific vasculature using NURBS and perform FSI analyses directly on these models. Templates for Horizontal Axis Wind Turbine (HAWT) blades and rotors were recently developed in [8], and employed in the FSI analysis in [9]. We would like to note that what we call "templates" are versions of what are called *features* in the design community. See [31] for a general discussion and [19] for a discussion of mechanical features for manufacturing.

In this paper we introduce a novel approach to NURBS solid modeling and representation for IGA. The method is based on a bottom-up, or patch-wise concept, where complex models are composed of relatively simple components. Our method is founded on the template-based modeling concept and integrates paradigms such as surface expansion and parametrization extraction. We extend the template modeling concept by employing an operator-based representation, which is similar to the concept of generative or procedural modeling [23,20]. That is, a complex geometrical model is not described in terms of its final data structures. Instead, one describes the modeling operations that lead to the final geometry. We adapt this concept in order to describe the evolution of a NURBS model, starting from a simple initial shape or a collection of such shapes. The modeling operations and the basic shapes can be expressed in terms of modeling commands and templates. This results in a drastic reduction of model size. It furthermore provides a basis for model adaptions on multiple levels, as all templates and modeling operations are inherently parametric.

This article is structured as follows: the mathematical foundations of NURBS solids are described in Section 2. Section 3 describes the details of our modeling approach and gives some implementation remarks. Section 4 presents the application of our methodology to the modeling the geometry of Vertical Axis Wind Turbines (VAWT). The resulting NURBS model may be used in the isogeometric FSI simulations. We show a preliminary moving-domain computation of our VAWT design under realistic wind conditions and rotation speeds. A more detailed study of VAWTs is planned in the future work. Section 5 presents discussion and conclusions, and outlines future research directions.

2. NURBS solids

NURBS-based geometric representations employ a set of parametric functions for describing an object. The basic elements for setting up these functions are the knot vectors. A knot vector is a collection of real-valued knots, ξ_i, which subdivide a one-dimensional parametric domain $\Omega_B^1 := [\xi_a, \xi_b] \subset \mathbb{R}$ into elements. For our purposes we focus on *open knot vectors* that are always of the form

$$\Xi := \{\underbrace{\xi_a, \ldots, \xi_a}_{p+1}, \xi_{p+1}, \ldots, \xi_n, \underbrace{\xi_b, \ldots, \xi_b}_{p+1}\}, \tag{1}$$

that is, both the lower boundary ξ_a and the upper boundary ξ_b are repeated $p + 1$ times at their respective position.

Knot vectors are the basis for evaluating B-Spline basis functions, typically using the Cox-De Boor formulas, the details of which may be found in the standard texts on NURBS or in the growing number of articles dealing with IGA [26,38,39]. We wish to stress two details. Firstly, open knot vectors yield B-Spline basis functions that are interpolatory at ξ_a and ξ_b, which greatly simplifies imposition of boundary conditions in simulations. Secondly, open knot vectors encode in their form the number $(n + 1)$ and degree (p) of the B-Spline basis functions $N_i^p(\xi) : \Omega_B^1 \to [0, 1]$, where $i = 0, \ldots, n$.

In order to represent solids, trivariate B-Spline functions $N_{ijk}^{pqr}(\xi, \eta, \zeta) : \Omega_B^3 \to [0, 1]$ are employed. They may be computed as tensor products of univariate B-Splines as

$$N_{ijk}^{pqr}(\xi, \eta, \zeta) = N_{ijk}^{pqr}(\boldsymbol{\xi}) = N_i^p(\xi) N_j^q \eta) N_k^r(\zeta), \tag{2}$$

where $N_j^q(\eta)$ and $N_k^r(\zeta)$ are computed from knot vectors $\mathbf{H}$ and $\mathbf{Z}$, respectively. The parametric domain Ω_B^3 of the trivariate B-Spline basis functions becomes

$$\Omega_B^3 := \Xi \times \mathbf{H} \times \mathbf{Z} = [\xi_a, \xi_b] \times [\eta_a, \eta_b] \times [\zeta_a, \zeta_b]. \tag{3}$$

We note that the trivariate B-Spline basis functions $N_{ijk}^{pqr}(\boldsymbol{\xi})$ are completely determined by a set of three open knot vectors Ξ, $\mathbf{H}$, and $\mathbf{Z}$.

Using the basis functions $N_{ijk}^{pqr}(\boldsymbol{\xi})$ we can now define a B-Spline solid as the set of all points $\mathbf{x} \in \mathbb{R}^3$ that result from the mapping