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Material point method for dynamic analysis of saturated porous media under external contact/impact of solid bodies

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ABSTRACT

With the use of the u–p form governing equations of saturated porous media, the coupling material point method (CMPM) is developed to predict the dynamic responses of saturated soil. The contact/impact problem between saturated porous media and solid bodies, like soil–structure interaction, is solved under the framework of the MPM. In the proposed strategy, the dynamic analysis of saturated soil and solid bodies is handled by the CMPM and the original MPM respectively. The interaction between saturated soil and solid bodies is simulated by a new contact algorithm which effectively avoids the interpenetration between saturated soil and solid bodies. The proposed approach circumvents the difficulties associated with the conventional spatial discretization methods such as the finite element method in simulating the contact/impact behavior between saturated soil and solid bodies. Representative examples are used to verify the proposed model-based simulation procedure and to demonstrate its potential in impact analysis related to geotechnical engineering.

1. Introduction

An understanding of dynamic behavior of saturated porous media is of great interest in geotechnical engineering field. Owing to the critical relevance to foundation engineering under various kinds of structural, soil and loading conditions, the soil–structure interaction has been a subject of active research for many years, like the saturated soil deposit under various environmental loadings. The framework of the dynamic analysis of saturated porous media was first established by Biot [5,6]. However, people could not carry out the numerical computation of the problem until the Finite Element Method (FEM) came into being. With the use of the u–w–p, u–U and u–p form governing equations, a variety of FEMs, such as the work by Prevost [18], Zienkiewicz and Shiomi [33], Lewis and Schrefler [13] and Borja et al. [7] have been developed for the solid–fluid coupled problems in the past decades. These methods can be used to deal with the complex geometry, inhomogeneity and nonlinear behavior of saturated porous materials.

The FEM, as a robust spatial discretization method for the analysis of a wide range of mechanical problems, however, also exhibits some disadvantages in some special problems, particularly when mesh distortion occurs in the large deformation and failure analysis. In order to avoid the deficiencies of the standard FEM, alternative spatial discretization methods such as "meshless" or "meshfree" methods have been proposed and developed by the research community in the recent years, among which the material point method (MPM) is a representative one.

The development of the MPM can be traced back to the work of Harlow [11], which studied fluid flow with the use of material points moving through a fixed spatial grid. Sulsky et al. [20] first extended the MPM from fluid dynamics to solid dynamics problems, and Sulsky et al. [24] further developed the MPM to simulate the problems such as contact/impact, penetration and perforation with history-dependent internal state variables, without invoking any master/slave relationship as required by the FEM. In the past decade, the MPM has evolved into a robust spatial discretization method capable of handling many challenging engineering problems, as demonstrated by the representative papers by Sulsky and Schreyer [22], York and Sulsky [27], Nairn [17], Chen et al. [8], Love and Sulsky [15,16], Sulsky and Schreyer [23], Zhang et al. [32], Zhang and Zou [29], Steffen et al. [19], and Wallstedt and Guilkey [25]. In fact, the MPM can also be regarded as the FEM formulated in an arbitrary Lagrangian–Eulerian framework, which utilizes two kinds of meshes, one is the material or Lagrangian mesh defined over material domain under consideration, and the other is the spatial or Eulerian mesh defined over computational domain. Due to the similarity between the FEM and the MPM, the interface between these two methods could be easily established for large-scale model-based simulations. Recently, several alternative forms from the original MPM as proposed by Sulsky et al. [20] have been developed. Guilkey and Weiss [10]

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and Sulsky and Kaul [21] proposed an implicit formulation of the MPM, respectively, Bardenhagen and Kober [4] developed a generalized interpolation MPM. In geotechnical engineering field, some problems have been solved using the MPM, such as Coetzee and Vermeer [9] and Zhou et al. [31], in which soil is considered as a kind of single-phase materials, and the interaction between solid skeleton and pore fluid in soil is not taken into account. In the present paper, the coupling material point method (CMPM), as an extension of the original MPM, is developed to predict the dynamic responses of saturated soil subject to contact/impact, based on the *u*–*p* form governing equations.

Contact phenomena are widely observed in engineering fields. Contact computation is remarkably difficult, due to the complexity of the physics and the mathematics involved, as discussed in the review article by Barber and Ciavarella [1]. Because a single-valued velocity field is used for updating the positions of material points, the no-slip (or sticking) contact between two different bodies can be handled automatically at no additional cost using the original MPM, and the contact surface need not to be detected. Furthermore, Bardenhagen et al. [2] and Bardenhagen and Guilkey [3] extended the original MPM to the friction (or slip) contact between deformable solid bodies, which allows Coulomb friction and slip at contact nodes.

In geotechnical systems including structure and saturated soil foundation, the dynamic analysis is more complicated, in which there are two kinds of coupled problems: (1) the coupling between solid skeleton and pore fluid in saturated soil, and (2) the contact coupling between structure and soil foundation. The first problem can be solved by the FEM and the CMPM as described later. The remaining issue is how to handle the second problem. Lewis and Tran [14] introduced a contact element in the consolidation problems, which allows the relative movement between saturated soil and structure. Zhang [30] used substructure technique to simulate saturated soil–structure interaction. Some analytical solutions about the pile–soil interaction were developed by Zeng and Rajapakse [28], Jin et al. [12] and Wang et al. [26]. In this paper, an alternative contact algorithm is developed within the framework of the MPM to simulate the contact behavior between solid bodies and saturated soil, in which the relative movement between solid bodies and saturated soil is allowed, following Coulomb friction law. The behavior of solids and saturated soil in the contact system is simulated by the original MPM and the CMPM, respectively.

The outline of the remaining sections in the paper is as follows. First, the governing equations of saturated soil are introduced, and the *u*–*p* form governing equations are adapted to derive the formulation of the CMPM. Next, the discrete form of the governing equations is given in detail. In the present work, the explicit time integration scheme is used in the CMPM to solve the interaction between solid skeleton and pore fluid in saturated soil. The coupled problem between structure and soil foundation is handled by the proposed contact algorithm as described in Section 4, in which the penetration of pore fluid and solid skeleton in saturated soil to solid bodies at the contact surface is not allowed. Finally, representative numerical examples are given to demonstrate the validity of the proposed simulation procedure.

2. Governing equations of saturated soil

The governing equations of fully saturated soil can be found from the general case dealt with in Lewis and Schrefler [13] with the assumption of isothermal behavior. These equations can be written as

$$\sigma_{ij,j} + \rho g_i - \rho \ddot{u}_i - \rho_f \dot{w}_i = 0, \tag{1a}$$

$$-p_{,i} + \rho_f g_i = \rho_f(\ddot{u}_i + \dot{w}_i/n) + k^{-1} w_i \gamma_f, \tag{1b}$$

$$Q(\alpha \dot{u}_{i,i} + w_{i,i}) + \dot{p} = 0, \tag{1c}$$

in which the first equation describes the linear momentum balance of the multi-phase medium, the second one is the linear momentum balance equation of pore fluid (Darcy's equation) and the third one is the mass balance equation of pore fluid. The convective terms are neglected in the above equations. In Eqs. (1a–c), $\sigma_{ij} = \sigma''_{ij} - \alpha p \delta_{ij}$ means the total stress, where σ''_{ij} is the effective stress tensor, p the pore pressure. u_i the displacement of solid skeleton, w_i the average pore fluid velocity relative to the solid skeleton. g_i the body force. α is Biot's constant, k fluid permeability, fluid bulk weight $\gamma_f = \rho_f g_r$, in which g_r is gravity. $Q = [(\alpha - n)/K_s + n/K_f]^{-1}$, with K_s and K_f, the bulk moduli of solid skeleton and pore fluid. The density of the mixture can be expressed as $\rho = n\rho_f + (1-n)\rho_s$, where n is the porosity of saturated soil, ρ_s represents the density of solid skeleton, ρ_f the density of pore fluid.

Neglecting the apparently small terms in Eqs. (1a–c) as mentioned in Lewis and Schrefler [13], the *u*-*p* form governing equations of saturated porous media can be obtained as follows:

$$\sigma_{ij,j} + \rho g_i - \rho \ddot{u}_i = 0, \tag{2a}$$

$$\dot{p} = -Q(k\gamma_f^{-1} R^e_{i,i} + \alpha \dot{u}_{i,i}), \tag{2b}$$

where

$$R^e_i = -p_{,i} + \rho_f g_i - \rho_f \ddot{u}_i. \tag{3}$$

The first one of Eqs. (2a) and (2b) is still the linear momentum balance equation for the mixture of saturated porous media, while the second one is interpreted as the variation rate of pore pressure in the current research.

The initial conditions for the dynamic analysis of saturated soil can be expressed as

$$u_i = u^0_i, \quad \dot{u}_i = \dot{u}^0_i, \quad p = p^0 \text{ at } t = 0, \tag{4}$$

which are in general obtained by means of a preliminary static solution to guarantee the satisfaction of the governing equations at $t = 0$.

Governing equations are solved numerically in the region Ω, the boundary conditions in the displacement field are given as

prescribed displacement:

$$u_i = \bar{u}_i \text{ on } \partial\Omega_u, \tag{5a}$$

and

external traction:

$$\sigma_{ij} n^b_{j,\tau} = \tau_i \text{ on } \partial\Omega_\tau, \tag{5b}$$

where $\mathbf{n}^b_\tau$ is the normal to the boundary $\partial\Omega_\tau$. The boundary conditions in the pore pressure field are written as

prescribed pore pressure:

$$p = \bar{p} \text{ on } \partial\Omega_p, \tag{6a}$$

and

fluid flux:

$$q_n = n^b_{i,q} R^e_i \cdot k/\gamma_f \text{ on } \partial\Omega_q, \tag{6b}$$

where $\mathbf{n}^b_q$ is the normal to the boundary $\partial\Omega_q$.

Accompanied by the appropriate initial and boundary conditions, as well as the proper constitutive laws, the dynamic responses of saturated soil can be predicted by the CMPM, which will be prescribed in the next section.

3. CMPM for saturated porous media

As compared with the original MPM, a similar procedure for the spatial discretization is used to develop the CMPM for the dynamic analysis of saturated soil. As shown in Fig. 1, two kinds of spatial discretizations are used in the MPM. First, the initial configuration