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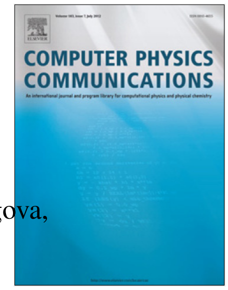

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Anderson acceleration method of finding steady-state particle size distribution for a wide class of aggregation-fragmentation models

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Abstract

A fast numerical method of finding steady-state distributions of particles sizes for a wide class of aggregation-fragmentation models, including the models with a source of monomers is developed. The method is based on a fast evaluation scheme for large sets of non-linear Smoluchowski-type ODE with an application of the Anderson acceleration method of the fixed point iterations. In the numerical tests of the suggested approach we demonstrate that huge sets of non-linear ODE may be solved with high precision in terms of Euclidian norm of the residual in modest times with use of a regular desktop computer. We compare our numerical solutions with the known analytical results for the steady-state distributions as well as with the other fast numerical schemes and prove the high accuracy of the novel method and its significant superiority with respect to the existing fast numerical method of solution of the addressed problems.

Keywords: aggregation, fragmentation, Smoluchowski equation, low-rank matrices, Anderson acceleration, steady-state distributions

1 Introduction

The processes of aggregation and fragmentation are ubiquitous in nature and occur at all length and time scales [25], from the nano-scales of molecular aggregation and fragmentation [11,18,35], to the astronomical scales, where aggregation of dust in interstellar dust clouds and proto-planetary discs [5,8,10,44] as well as collisional fragmentation of celestial bodies (comets, meteoroids or planets) takes place, see e.g. [2]. Airborne particles perform Brownian motion in atmosphere and coalesce giving rise to smog droplets [25,40]. Aggregation and fragmentation also occurs in a social life: for instance, consolidation of users in the Internet leads to the emergence of internet communities, forums, which can further merge or split [14, 23]. Vortexes in a fluid flow merge and decompose forming turbulent cascades [3,49].

There exists a wide class of systems, where a balance between aggregation and fragmentation is established, leading to a steady-state distribution of objects size, so that, in spite of permanent coalescence and breakage of the system elements, the average concentration (or number) of elements of a particular size does not alter with time. Such systems may be exemplified by "living" polymers solutions, e.g. , in vivo solutions of prions (cell proteins), which can aggregate in large fibrils destroying cellular membrane – the malicious process taking place in Alzheimer-like diseases. At some conditions a steady-state distribution of the fibril sizes may be achieved [35]. Another example is the planetary rings, e.g. , Saturn Rings, comprised of billions of icy particles. Here a subtle equilibrium between fragmentation of particles colliding with large impact velocities and aggregation of particles

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