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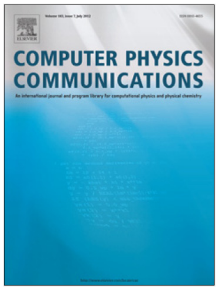

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Adaptive time-stepping Monte Carlo integration of Coulomb collisions

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Abstract

We report an accessible and robust tool for evaluating the effects of Coulomb collisions on a test particle in a plasma that obeys Maxwell-Jüttner statistics. The implementation is based on the Beliaev-Budker collision integral which allows both the test particle and the background plasma to be relativistic. The integration method supports adaptive time stepping, which is shown to greatly improve the computational efficiency. The Monte Carlo method is implemented for both the three-dimensional particle momentum space and the five-dimensional guiding center phase space.

Detailed description is provided for both the physics and implementation of the operator. The focus is in adaptive integration of stochastic differential equations, which is an overlooked aspect among existing Monte Carlo implementations of Coulomb collision operators. We verify that our operator converges to known analytical results and demonstrate that careless implementation of the adaptive time step can lead to severely erroneous results.

The operator is provided as a self-contained Fortran 95 module and can be included into existing orbit-following tools that trace either the full Larmor motion or the guiding center dynamics. The adaptive time-stepping algorithm is expected to be useful in situations where the collision frequencies vary greatly over the course of a simulation. Examples include the slowing-down of fusion products or other fast ions, and the Dreicer generation of runaway electrons as well as the generation of fast ions or electrons with ion or electron cyclotron resonance heating.

Keywords: Coulomb collision, Monte Carlo, Fokker-Planck equation, Milstein method

PROGRAM SUMMARY

Program Title: AMCC – (A)daptive (M)onte-(C)arlo (C)oulomb collisions
Program Files doi: `http://dx.doi.org/10.17632/wd3t3t6dy7.1`
Licensing provisions: MIT licence
Programming language: Fortran 95
Nature of problem: Test-particle tracing is a common feat within existing fusion applications. While efficient adaptive methods exist for integrating the incompressible Hamiltonian flow, the effects of Coulomb collisions are commonly implemented with far less sophisticated algorithms.
Solution method: The relativistic Fokker-Planck equation for test-particles in Maxwell-Jüttner background plasmas is converted into a stochastic differential equation. The stochastic differential equation is solved using adaptive Monte Carlo techniques. Methods to evaluate the effect of Coulomb collisions for both the three-dimensional particle momentum space and the five-dimensional reduced guiding center phase space are included.
Additional comments including restrictions and unusual features:
The package includes optionality to evaluate the relativistic Fokker-Planck coefficients, a feature useful for constructing accurate orbit averaged collision operators. The package also provides explicit one-step sympletic integrator for the relativistic Lorentz force that can be used for tracing test-particles in given electromagnetic backgrounds.

1. Introduction

Standard Runge-Kutta methods developed for ordinary differential equations (ODEs), are based on first converting the ODE into an integral equation, and then discretizing the integral. Stochastic differential equations (SDEs), often encountered when stochastic processes such as collisions are of interest, can equivalently be transformed into integral equations, with the exception that the resulting integral is non-Riemannian [1]. The discretization rules used for ODEs then no longer apply and, instead, either Itô or Stratonovich calculus must be adopted to obtain numerical methods for solving SDEs.

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