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Correct averaging in transmission radiography: Analysis of the inverse problem

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Transmission radiometry is frequently used in industrial measurement processes as a means to assess the thickness or composition of a material. A common problem encountered in such applications is the so-called dynamic bias error, which results from averaging beam intensities over time while the material distribution changes. We recently reported on a method to overcome the associated measurement error by solving an inverse problem, which in principle restores the exact average attenuation by considering the Poisson statistics of the underlying particle or photon emission process. In this paper we present a detailed analysis of the inverse problem and its optimal regularized numerical solution. As a result we derive an optimal parameter configuration for the inverse problem.

1. Introduction

Transmission radiometry is a widely applied measurement principle for thickness, density or composition of materials [1–3]. It is very often applied to material flows, e.g. for measurement of thickness in paper production or metal sheet rolling, but also for composition measurement on multiphase flows in the oil and gas industry. These flows differ from quasi-static to fast-changing regarding the frequency and gradients of flow thickness or density. A simplified setup of a transmission radiometry sensor configuration is shown in Fig. 1. It comprises a radiation source, which emits photons or particles, and a radiation detector, which counts the photons or particles after they have passed the material under investigation. Additional means of beam collimation are helpful to form only a thin radiation beam and to prevent scattered radiation entering the detector. Let us assume that the particle flux of the source has a constant average value. Further let $\langle N\rangle$ denote the number of particles detected in a given time interval when there is material between source and detector and $\langle N_0\rangle$ the reference count number if there is no material between source and detector. From physical consideration it follows, that the radiation is exponentially attenuated in the material, that is

$$\langle N\rangle = \langle N_0\rangle \exp(-A). \quad (1)$$

For mono-energetic radiation the total attenuation A depends linearly on material thickness d and density ρ, that is

$$A = \mu_p \rho d \quad (2)$$

with μ_p denoting the mass attenuation coefficient. The particle or photon emission at the source is a statistical process as well as the detection. The probability that the detector registers N counts in a time interval for which $\langle N\rangle$ counts are being expected is given by the Poisson distribution

$$p(N) = \frac{\lambda^N}{N!} \exp(-\lambda) \quad (3)$$

with $\lambda = \langle N\rangle$ being the expectation value. The Poisson distribution has the following properties:

(1) The standard deviation is $\sigma = \sqrt{\lambda}$.
(2) For small values of λ the distribution has an increasingly higher skewness.
(3) According to the central limit theorem the distribution approaches a symmetric normal distribution for large λ.

From property (1) it follows, that statistical accuracy of a single measurement can be improved by increasing $\langle N\rangle$. Since in industrial measurement applications the source strength should be kept low for reasons of radiation protection and hazard reduction it follows that only increasing the counting interval length would then

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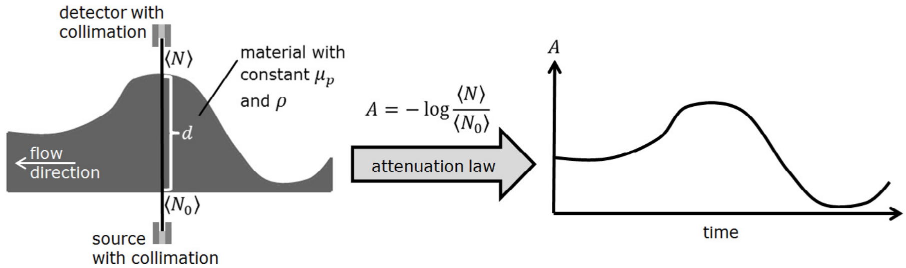

Fig. 1. Principle of a radiation transmission measurement. Radiation emitted from the collimated source passes the material (with mass attenuation coefficient μ_p and density ρ) under investigation and is registered by the collimated detector.

be an appropriate measure. However, if the material distribution changes within that interval, an error in the total attenuation results, as we will show below.

2. Averaging methods

For the following analysis it is convenient to consider the counting of the detector as a discrete averaging procedure. Let T denote a longer time interval, for which the expected count rate is considered as sufficient for a qualitatively good measurement. Further assume, that during T the material distribution, e.g. the material thickness, and thus A changes significantly. Now we may assume, that we can subdivide the interval T into n shorter time intervals of duration T_S and that A can be considered as constant during T_S. If we were to make measurements during T_S, which we will further refer to as instantaneous measurements, we would find, that the count rate in each interval T_S has a high uncertainty due to property (1) above. The conventional (and wrong) way of averaging is to let the detector count all arriving particles or photons within T and then to compute an average attenuation

$$\tilde{A} = -\log\frac{\overline{N}_{arithmetic}}{\langle N_0\rangle} \tag{4}$$

from the arithmetic mean value

$$\overline{N}_{arithmetic} = \frac{1}{n}\sum_{k=1}^{n} N_k. \tag{5}$$

Let us now denote the expectation value of an instantaneous measurement as $\langle N\rangle_k$. Then it immediately follows, that the instantaneous total attenuation is

$$A_k = -\log\frac{\langle N\rangle_k}{\langle N_0\rangle} \tag{6}$$

and hence the 'true' average is given by

$$\begin{aligned}\overline{A} &= \frac{1}{n}\sum_{k=1}^{n} A_k = -\frac{1}{n}\sum_{k=1}^{n}\log\frac{\langle N\rangle_k}{\langle N_0\rangle}\\ &= -\log\sqrt[n]{\prod_{k=1}^{n}\frac{\langle N\rangle_k}{\langle N_0\rangle}} \approx -\log\frac{\overline{\langle N\rangle}_{geometric}}{\langle N_0\rangle}.\end{aligned} \tag{7}$$

Here, a simple substitution of the unknown $\langle N\rangle_k$ by the actually registered events N_k is not possible in general. In case of low count rates $\langle N\rangle_k$, the absence of registered photons during at least one time interval T_S is likely because of the Poisson distributed photon emission process. Such a single event $N_k = 0$ would abort the calculation of $\overline{A}$. Therefore, the conventional method (4) is a common approach for averaging since it handles this problem. For a comparison between the conventional method and the true average, let us assume we have a large number of instantaneous measurements n such that $\overline{N}_{arithmetic} \approx \overline{\langle N\rangle}_{arithmetic} = \frac{1}{n}\sum_{k=1}^{n}\langle N\rangle_k$. In that case the average attenuation calculated by (4) is an underestimation,

$$\tilde{A} \le \overline{A}, \tag{8}$$

because of the inequality of means: $\overline{\langle N\rangle}_{arithmetic} \ge \overline{\langle N\rangle}_{geometric}$. The equality only holds for constant $\langle N\rangle_k$, whereas the deviation increases with increasing variability of the flow. This effect is presented in our previous work [4]. This so-called dynamic bias error appears in void fraction measurement for two-phase flows for instance. In case of slug flow and turbulent flow the amount of attenuating water that passes the gamma-rays strongly varies within short time intervals which leads to a significant overestimation of the void fraction [5]. Further analysis of the dynamic bias error can be found in [6–10]. A correction method is given in [10] which bases on a momentum expansion of the count rate distribution. Further, correction methods for the dynamic bias error in two-phase flow measurements are also proposed in [5,11–13]. The first also takes the Poisson distribution into account in order to perform a first-order correction.

As an alternative, we introduced in [4] a new method for correct averaging which we outline in the following. The probability $p(N)$ of registering N particles at the detector for expectation values $\langle N\rangle$ is given by the Poisson distribution (3). With that, one obtains the conditional probability distribution $p(N\,|\,A)$ of registering N events at the detector for a given attenuation A by

$$p(N\,|\,A) = \frac{(N_0\exp(-A))^N}{N!}\exp(-N_0\exp(-A)). \tag{9}$$

Assuming the probability density $\theta(A)$ for occurrence of A in a flow to be known, the average attenuation is given by

$$\bar{A} = \int_0^\infty A\theta(A)\mathrm{d}A. \tag{10}$$

In order to obtain $\theta(A)$, one can apply the law of total probability to the introduced distributions, which leads to

$$f(N) = \int_0^\infty \theta(A)p(N\,|\,A)\mathrm{d}A, \tag{11}$$

with $f(N)$ as the frequency distribution of the count rates N. In radiation densitometry, these count rates are observed (and so $f(N)$) and the attenuation is unknown (and so $\theta(A)$). Therefore, radiation densitometry inherently presents an inverse problem and solving (11) becomes the main part of the correct averaging method.

3. Analysis of the discrete system

Since an analytical solution of Eq. (11) cannot be obtained in general, a discrete version is required. In practice the registered detection events are integer values. Therefore, $f(N)$ is already a discrete distribution. For numerical treatment it should also be finite, hence we limit $f(N)$ to a finite set of length L. Further, one can