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Isogeometric simulation of Lorentz detuning in superconducting accelerator cavities

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ABSTRACT

Cavities in linear accelerators suffer from eigenfrequency shifts due to mechanical deformation caused by the electromagnetic radiation pressure, a phenomenon known as Lorentz detuning. Estimating the frequency shift up to the needed accuracy by means of standard Finite Element Methods, is a complex task due to the non exact representation of the geometry and due to the necessity for mesh refinement when using low order basis functions. In this paper, we use Isogeometric Analysis for discretizing both mechanical deformations and electromagnetic fields in a coupled multiphysics simulation approach. The combined high-order approximation of both leads to high accuracies at a substantially lower computational cost.

1. Introduction

Controlling the resonant frequency of cavity eigenmodes in a particle accelerator is crucial in order to guarantee the synchronization of the electromagnetic wave and the particle bunches. Such frequency is determined essentially by the geometry of the cavity walls, which is therefore a critical parameter for the design of the cavity. The high-energy electromagnetic field inside the cavity exerts a radiation pressure on the walls, which causes a mechanical deformation of the geometry. Albeit small, this deformation may lead to a significant shift of the resonant frequency. This effect, known as *Lorentz detuning* [1–4], needs to be predicted with high precision in order to achieve a robust cavity design.

Standard Finite Element Methods (FEM) may require an extremely high level of mesh refinement to achieve sufficient accuracy when evaluating Lorentz detuning, due to inaccuracies when approximating the deformed and undeformed cavity walls in the FEM mesh and due to the limited accuracy of typical low-order FEM basis functions. In this work, we propose a simulation strategy based on Isogeometric Analysis (IGA) [5] which allows an exact representation of the geometry and the direct application of the computed deformation to the starting geometry, without any further approximation. Finally it offers the possibility to accurately approximate the electromagnetic fields using high-order elements [6].

The outline of this paper is as follows: first we introduce the coupled electromagnetic–mechanical model describing Lorentz detuning. In the subsequent section Isogeometric Analysis is introduced along with an overview on the particular discretization used for Maxwell's equations. Finally we present the results obtained for the standard cylindrical test case and for the TESLA cavity geometry [7].

2. Multi-physics model for Lorentz detuning

Consider a one cell cavity geometry as the one depicted in Fig. 1. Let the two disjoint open domains with Lipschitz continuous boundaries $\Omega_W \subseteq \mathbb{R}^3$ and $\Omega_C \subseteq \mathbb{R}^3$ represent the cavity walls and the interior of the cavity, respectively. Let $\Gamma_{CW} = \overline{\Omega}_C \cap \overline{\Omega}_W$ denote the interface between the two domains. To evaluate the frequency shift, it is necessary to solve Maxwell's eigenproblem inside the undeformed and deformed cavity and an elasticity problem in the cavity walls. We employ linear elasticity theory since the deformations are very small. The radiation pressure on the common interface Γ_{CW} introduces a coupling between the two problems [8]. The calculation steps are as follows:

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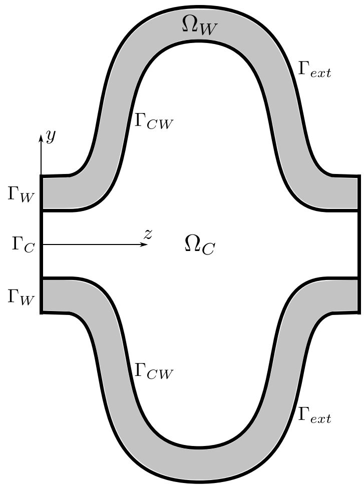

Fig. 1. 2D cut of the 3D computational domain for simulating Lorentz detuning in one cell of the TESLA cavity [7] (not to scale) and labels for the domains and the boundaries (*yz* section). The full cell is the result of a revolution around the *z* axis.

Step 1. Solve Maxwell's eigenproblem in Ω_C:

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{E} \right) = \omega_0^2 \epsilon_0 \mathbf{E} \quad \text{in } \Omega_C \tag{1a}$$

with the boundary conditions

$$\begin{cases} \mathbf{E} \times \mathbf{n}_c = 0 & \text{on } \Gamma_{CW} \\ \left(\frac{1}{\mu_0} \nabla \times \mathbf{E} \right) \times \mathbf{n}_c = 0 & \text{on } \Gamma_C \end{cases} \tag{1b}$$

where μ_0 and ϵ_0 are the permeability and permittivity of vacuum and $\mathbf{n}_c$ is the outward unit normal to Ω_C. We assume time-harmonic fields with $\mathbf{E}$ a phasor given in terms of peak values. As cavity walls are often composed of a superconducting material, *e.g.* niobium, in order to reduce losses, they are assumed here to behave as a perfectly conducting boundary. At the two irises Γ_C, a Neumann condition is enforced, which is a common approximation corresponding to assuming the cell to be one of an infinite chain of cells. The eigenmode solution delivers a number of eigenfunction–eigenvalue couplets, corresponding to the possible modes within the cavity. The accelerating mode of interest is the first transverse magnetic mode (TM_{010}). Let $\mathbf{E}_0$ be the computed electric field and ω_0^2 the corresponding eigenvalue, then $f_0 = \frac{\omega_0}{2\pi}$ is the resonant frequency for the accelerating eigenmode in the undeformed geometry.

Step 2. Compute the magnetic field $\mathbf{H}_0$ for the first accelerating eigenmode as

$$\mathbf{H}_0 = \frac{i}{\omega_0 \mu_0} \nabla \times \mathbf{E}_0. \tag{2}$$

The accelerating mode exerts on the cavity walls a radiation pressure with one component at 0 frequency and one component at frequency 2 f_0. In practice, the latter can be neglected and the radiation pressure on Γ_{CW} is approximated by a time-constant value that may be expressed as

$$p = -\frac{1}{4} \epsilon_0 \left(\mathbf{E}_0 \ \mathbf{n}_c \right) \cdot \left(\mathbf{E}_0^* \ \mathbf{n}_c \right) + \frac{1}{4} \mu_0 \left(\mathbf{H}_0 \times \mathbf{n}_c \right) \cdot \left(\mathbf{H}_0^* \times \mathbf{n}_c \right) \tag{3}$$

where $\mathbf{E}_0$ and $\mathbf{H}_0$ are peak values and $(\cdot)^*$ denotes the complex conjugate.

Step 3. Solve the following linear elasticity problem in the walls domain Ω_W

$$\nabla \cdot \left(2\eta \nabla^{(S)} \mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u} \right) = 0 \quad \text{in } \Omega_W \tag{4a}$$

with boundary conditions

$$\begin{cases} \mathbf{u} = 0 & \text{on } \Gamma_W \\ \left(2\eta \nabla^{(S)} \mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u} \right) \ \mathbf{n}_w = p \ \mathbf{n}_w & \text{on } \Gamma_{CW} \\ \left(2\eta \nabla^{(S)} \mathbf{u} + \lambda \mathbf{I} \nabla \cdot \mathbf{u} \right) \ \mathbf{n}_w = 0 & \text{on } \Gamma_{ext} \end{cases} \tag{4b}$$

for the displacement $\mathbf{u}$. In (4) we denote by $\nabla^{(S)}$ the symmetric gradient, while η and λ are the Lamé parameters of the wall constituent material and $\mathbf{n}_w$ is the outward unit normal to Ω_W. On Γ_{CW} the radiation pressure p is applied.

Step 4. Let the deformed walls domain Ω_W' be defined as

$$\Omega_W' \equiv \{ \mathbf{x} + \mathbf{u}(\mathbf{x}) \, , \ \mathbf{x} \in \Omega_W \} \, , \tag{5}$$

and the deformed cavity boundary Γ_{CW}' as

$$\Gamma_{CW}' \equiv \{ \mathbf{x} + \mathbf{u}(\mathbf{x}) \, , \ \mathbf{x} \in \Gamma_{CW} \} \, . \tag{6}$$

Furthermore, let Ω_C' denote the domain enclosed by Γ_{CW}' and Γ_C.

Step 5. Solve Maxwell's eigenproblem in Ω_C':

$$\nabla \times \left(\frac{1}{\mu_0} \nabla \times \mathbf{E}' \right) = \left(\omega_0' \right)^2 \epsilon_0 \mathbf{E}' \quad \text{in } \Omega_C'$$

with the boundary conditions

$$\begin{cases} \mathbf{E}' \times \mathbf{n}_c' = 0 & \text{on } \Gamma_{CW}' \\ \left(\frac{1}{\mu_0} \nabla \times \mathbf{E}' \right) \times \mathbf{n}_c' = 0 & \text{on } \Gamma_C' \end{cases}$$

and let $\left(\left(\omega_0' \right)^2 , \mathbf{E}_0' \right)$ denote the accelerating eigenmode. The shifted frequency is finally obtained as

$$f_0' = \frac{\omega_0'}{2\pi}$$

and the frequency shift due to Lorentz detuning as

$$\Delta f_0 = \left| f_0 - f_0' \right| . \tag{7}$$

This procedure can be carried out iteratively if necessary.

3. Numerical discretization

Isogeometric Analysis (IGA) was born, less than a decade ago [9], with the goal of *bridging the gap between Computer Aided Design (CAD) and Finite Element Method (FEM)*. The main distinctive feature of IGA is that CAD geometries, commonly defined in terms of Non-Uniform Rational B-splines (NURBS), are represented exactly throughout the analysis, regardless of the level of mesh refinement, while in standard FEM the computational domain needs to be remeshed when performing *h*-refinement and its geometry approaches the exact one only in the limit of vanishing mesh size *h*.

Moreover, in addition to *h*-refinement and *p*-refinement, *k*-refinement [5] was introduced as a combination of degree elevation and mesh refinement, yielding approximation spaces with higher regularity properties. *k*-refinement has the advantage of not increasing the number of degrees of freedom of the problem, but produces matrices with larger bandwidth.

The particular IGA scheme adopted in this work takes advantage of the benefits of different approaches for each of the different physical subproblems being considered. The computational domains Ω_W and Ω_C are both defined via geometric mappings constructed in terms of NURBS basis functions. In solving the mechanical subproblem (4) an isoparametric approach is adopted so that the computed (discrete) displacement is defined in terms of the same NURBS basis and therefore the domain deformation (5) is treated in a straight-forward way by a simple displacement of the control-points. In solving the Maxwell sub-problem (1), on the